Macroeconomic Effect of Credit and Labor Frictions∗

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Abstract

At the aggregate level, the recent three recessions break down Okun’s law, and are instead associated with jobless recovery, i.e., both payroll employment and unemployment rate fall behind output recovery. Meanwhile the average labor productivity (ALP) tends to be acyclical. At the disaggregate level, productivity dispersion widens for both incumbent and new-entry firms, and the share of employment by large firms decreases in recessions. To this end, we construct an analytically tractable heterogeneous-entrepreneur model with both financial friction and search friction. All the shocks to aggregate productivity, to financial market and to labor market work through $TFP$ in general equilibrium. Persistence to these shocks is essential to support jobless recovery. On one hand, all these shocks share a similarly qualitative implication for dynamics of aggregate variables and shift the Beveridge curve through endogenous recruiting effort. However, only labor-market shock delivers acyclicality of $ALP$. On the other hand, the negative shock to aggregate productivity and financial market jointly generates a widening productivity dispersion and a decreasing share of employment by large firms in recession. Finally, we decompose the dynamic effect of credit friction on unemployment into two effects: reallocation channel and credit channel.

Key Words: Jobless Recovery, Beveridge Curve, Credit Friction, Labor Search Friction, Capital/Labor Misallocation, TFP.

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1 Introduction

At the macro level, in contrast with historical business cycles, the recent three recessions are followed by jobless recovery, i.e., both payroll employment and unemployment rate fall behind output recovery. Meanwhile the average labor productivity (ALP), as shown by Berger (2012), tends to be acyclical. At the micro level, productivity dispersion widens and the share of employment by large firms decreases in recessions. To this end, we construct an analytically tractable heterogeneous-entrepreneur model with both financial friction and search friction. All the shocks to aggregate productivity, to financial market and to labor market work through TFP in general equilibrium. Persistence to these shocks is essential to support jobless recovery. On one hand, all these shocks share a similarly qualitative implication for dynamics of aggregate variables except for ALP. Only the shock to labor market delivers acyclicality of ALP. On the other hand, only credit shock generates a widening productivity dispersion and a decreasing share of employment by large firms in recession. Finally, we decompose the dynamic effect of credit friction on unemployment into two effects: reallocation channel and credit channel.

Figure 1.1: Real GDP. Data Source: Bureau of Economic Analysis.

We start with some aggregate empirical facts. On the one hand, as standard in the literature, we document the output and the payroll employment in Figures 1.1 and 1.2 in turn. As shown in the first figure, output rebounded immediately after the rough, although it rebounded more quickly in the prior recessions. The employment numbers also recovered since the trough for the prior recessions, but they didn’t for the recent three recessions. What we want to emphasize is that the payroll employment is only one dimension of jobless recovery. As shown in Figure 1.3
in contrast to the prior recessions, high unemployment are also more persistent in the past three recessions. That is, Figures 1.2 and 1.3 jointly characterize the jobless recovery pattern of the past three recessions. A large number of the past research fails to address the unemployment pattern since their models assume a frictionless labor market.

Motivated by the above observations, we ask the following questions. What kind of the following shocks could drive jobless recovery, the shock to aggregate productivity, to credit market, or the labor market. Moreover, what is the potential role of shock persistence in explaining the different magnitude for the past three jobless recovery? What is the connection between credit imperfections and jobless recovery? Finally, what kind of shock generates predictions in line with both aggregate and micro empirical regularity?

Figure 1.2: Dimension I of Jobless Recovery: Non-farm Payroll Employment. Data Source: Bureau of Labor Statistics.

We construct a tractable model to address the above questions. In particular, we use this model to characterize and quantify the joint dynamic effect of credit crunch and labor frictions on jobless recovery with both capital and labor misallocation. There are two layers of frictions in the model. On the one hand, we model credit friction by using collateral constraint in the spirit of Kiyotaki and Moore (1997). The credit friction has been discussed extensively on its role for capital misallocation; see Buera, Kaboski and Shi (2011), Moll (2012) and Buera and Moll (2013) for example. They all assume a frictionless labor market. In real life, however, as well emphasized by Hall and Krueger (2012), “...The competitive market for a commodity, where all units are interchangeable and all trade for the same price, could hardly be a worse description of

1Alternatively, we can follow Koenders and Rogerson (2005) to summarize the data with HP filter.
the labor market. No Walrasian auctioneer determines the wage...” Instead, labor market analysis adopts random search or wage posting. On the other hand, labor misallocation emerges when the homogenous workers are subject to labor search. A larger volume of research after the seminal work on random search by Diamond, Mortensen and Pissarides. Moreover, Lagos (2006) builds on Mortensen and Pissarides (1994) to address the role of labor market friction for TFP distortion. We model the equilibrium unemployment via competitive (directed) search a la Moen (1997).

However, search model typically focuses on stationary distribution without capital accumulation. Some exceptions include Merz (1995) and Andolfatto (1996), both of which try to integrate search into standard RBC model. They are purely numerical.

Figure 1.3: Dimension II of Jobless Recovery: Unemployment Rate. Data Source: Bureau of Labor Statistics.

The baseline model is populated by two types of infinitely lived players in the economy, entrepreneurs and workers. There is a continuum of risk-neutral workers and each of them has one unit of identical and indivisible labor to supply. For simplicity, we assume workers make hand-to-mouth consumption. Only entrepreneurs have access to the credit market and the knowledge on production with capital and labor. Due to limited enforceability, capital misallocation emerges even in the absence of labor search frictions. Entrepreneurs are heterogeneous in net worth and receive idiosyncratic productivity shock. The distribution of the former is endogenous while that of

\footnote{It is equally tractable for us to alternatively use Diamond-Mortensen-Pissarides random search. There are two key reasons why we stick to competitive search. First, competitive search dispenses us with Nash bargaining, and instead delivers an endogenous wage scheme. Secondly, in contrast to random search, Hosios condition is always satisfied in competitive search. We can check that the main results are equally tractable if we resort to random search.}

\footnote{Non-trivial financial intermediary is introduced in the extension part.}
the latter is governed by some exogenous stochastic process. Entrepreneurs have to decide whether to post a wage contract in certain sub labor market. They can also choose to simply lend to other entrepreneurs. Even in the absence of the cost of wage posting, the implicit cost of producing is the interest rate in the credit market. In turn that in equilibrium there exists a cut-off point on productivity only above which entrepreneurs will choose to borrow, set up a wage scheme for workers and produce. The optimal wage schemes proposed by the active entrepreneurs is shown to generate an equilibrium wage dispersion. This is well in line with empirical work. See Mortensen (2003) for example. As standard in the literature on competitive search, workers in turn will be indifferent among entering various active sub labor markets.

In addition to the implication by heterogeneity at the micro level, our model also admits a tractable aggregation. We have fully characterization on both the stage equilibrium and the evolution on the distribution of productivity and net worth. We show that, it is the aggregate net worth, rather than the distribution, that really matters for general equilibrium. Moreover, we prove that, when the productivity distribution follows a Pareto distribution and the matching function in all sub labor markets admits a Cobb-Douglas form, then given the aggregate output is Cobb-Douglas. At the macro level, our model with both financial friction and labor search friction is shown to be equivalent to a heterogeneous-entrepreneurs model with financial friction but a Walrasian labor market.

The associated endogenous TFP is explicitly proved to be related to three pieces of aggregate shocks: to productivity, to collateral constraint in credit market and to matching efficiency in labor market. Then the aggregate net worth is shown to evolve like that in the classic Solow model, which makes the dynamics very tractable. In sum, all these three shocks work through the productivity wedge at the aggregate level in general equilibrium. A quantitative exercise with calibration suggests that these shock share very similar impulse responses for aggregate variables, such as output, TFP, unemployment, capital and interest rate. All these shock share the credit channel to affect the aggregate variable. That is, they exert dynamic influence on the aggregate net worth, which will be used by active entrepreneurs to match with labor by posting their optimal wage contract. The most essential findings on the quantitative exercise is that shock persistence matters. When the auto-correlation is very low, the pattern is jobless recovery tends to degenerate. That is, unemployment quickly decrease as the output rebounds. Besides, the magnitude on unemployment fluctuation is relatively small. On the contrary, when the persistence is very high, not only lag

4We are definitely not the first to reach the endogenous aggregate Cobb-Douglas production function. Related literature was initiated by the seminar work of Houthakker (1955) and mainly includes Butters (1977), Hall (1979), Jones (2005), Lagos (2006). Recent findings on aggregate production with TFP distortion are mainly set up with financial frictions. Main literature includes Buera, Kaboski and Shin (2011), Moll (2012) and Buera and Moll (2013).
between output and employment recovery widens, but also the shock effect on unemployment is relatively significant.

A natural question is, which shock is most essential driving the jobless recovery? As suggested in Figure 4.1, despite difference in quantitative magnitude, the aforementioned three shocks share each other with very similar implication for the dynamics of aggregate variables. Thus it is difficult, if possible, for us to identify the essential source for jobless recovery. Moreover, it is very likely that all these shocks work together in the past recessions. However, if we switch from the aggregate to micro level implication, then shock in credit market seems the most reasonable candidate. Here is the argument. The empirical finding by Kehrig (2011) reveals that, the productivity dispersion widens in recessions. Furthermore, he shows it is mainly due to the increasing proportion of unproductive firms. Through the lens of our model, the shock to either aggregate productivity or matching efficiency in labor market does not change the proportion of active entrepreneurs at all. However, the shock to credit market implies the proportion of active entrepreneurs and the aggregate output are negatively related, which is in line with both the micro and aggregate empirical regularity. The above argument also validates the merits with our setup involving heterogeneous rather than representative entrepreneurs.

The unemployment effect of credit friction is in twofold. One is through the credit channel, just as that by the aggregate productivity or matching efficiency shock. That is, the credit crunch worsens misallocation and in turn lowers the aggregate capital used to match with labor. The other one is through its reallocative effect among heterogeneous entrepreneurs. More specifically, given the aggregate net worth in any period, the shock to collateral constraint changes the composition of active entrepreneurs. What is associated with capital reallocation is the job creation and job destruction by different entrepreneurs. In a nutshell, it is the credit channel and the reallocation channel that jointly determine the dynamics by credit shock.

Even though the recent financial crisis has spawned a large volume of research addressing the role financial friction a la Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) in output fluctuation, relatively very few papers connect financial friction and unemployment. To the best of our knowledge, there are three pieces of exceptions. The first one is Wasmer and Weil (2004). They use matching function with random search in both credit market and labor market to model frictions prevalent in both markets. They show the interaction between credit and labor search frictions by taking a local approximation of non steady state. Since they stick to

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5The three papers we use to compare with ours are in all macro fields. Chodorow-Reich (2013) uses micro-level data to identify the employment effect of disruption in credit market.

the standard search framework, essentially there is no room for capital accumulation. Besides, it is very difficult to talk about global dynamics. Our paper not only preserves the tractability of labor search, but also admits capital accumulation and analytically characterize the global dynamics of capital accumulation. The second paper on financial and labor frictions is by Monacelli, Quadrini and Trigari (2011). Both their paper and mine model financial friction explicitly with limited commitment rather than reduced-form matching function. There are two pieces of main difference between our papers. One is that they adopt the standard search framework and thus similar to Wasmer and Weil (2004), firm size is constantly held at one. My paper instead allows us to address the role of credit and labor frictions on the distribution of firm size and its transitional dynamics. The other key difference is that, they mainly discuss the strategic use of debt by firms for the wage bargaining with workers while mine emphasize the interaction of credit and search frictions.

Our paper is most related to Buera, Fattal-Jaef and Shin (2013). Both of our papers comes up with a heterogeneous-entrepreneurs model with credit friction and labor search frictions. Both of our papers explore effect of credit crunch on unemployment. However, our paper also differ in several important dimensions. First, although our model involves in aggregate shocks to productivity, credit market and labor market, the constant return to capital revenue makes it still analytically tractable for both stage equilibrium and the transition dynamics. The high tractability clearly suggests that these three aggregate shocks work as productivity wedge via TFP at the aggregate level. Their model considers a very general framework with decreasing returns and occupational choice and therefore is basically numerical. Secondly, we adopt different modeling tricks to equilibrium unemployment. They specify a Walrasian labor market with a unique and publicly displayed price. However, only part of unemployed workers can have access to the labor market. We instead use a standard competitive search, in which active entrepreneurs propose their own optimal wage contract, which differ from each other. Finally, they focus on borrowing and lending within entrepreneurs while our paper allow for endogenous supply of external credit by financial intermediary. We also consider the case with heterogeneous leverage while their only consider a collateral constraint with the same leverage across heterogeneous agents.

The rest of the paper proceeds as below. Sections 2, Section 3 and Section 4 describes, characterizes and quantifies the baseline model in turn. Section 5 launches three pieces of model extension and Section 6 concludes. The appendix pools the proofs.

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7Garin (2012) extends Monacelli, Quadrini and Trigari (2011) by allowing for capital accumulation. Besides, Blanco, Mullins and Navarro (2013) extends the paper by introducing flexible number of employees and equilibrium default.
2 Model

2.1 Environment

Time is discrete and proceeds from zero to infinity. There are two types of infinitely lived players in the economy: workers and entrepreneurs. We assume there is no concern for occupational choice.

**Worker.** There is a representative household with $L$ measure of homogeneous household members. Each has one unit of indivisible labor. We assume the household has access to neither production nor credit market. If a worker is unemployed, she gets zero consumption. If a worker is matched with a entrepreneur, she receives a labor income. The household makes a hands-to-mouth consumption equally to every member by pooling wage revenues at the end of each period. The details on labor search and matching will be introduced later.

**Entrepreneur.** The measure of entrepreneurs is normalized to one. Only entrepreneurs have access to credit market and to the skills to manage capital for production. Entrepreneurs are heterogeneous in two dimensions, one is net worth $a$ while the other one is idiosyncratic productivity shock $\varphi$. The distribution of net worth is endogenously evolved over time while that of idiosyncratic shock is exogenous. There is no information asymmetry in the economy. Entrepreneur’s objective function of is given as below.

$$U_e = \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \cdot \log(d_t) \right],$$

where $d_t$ denotes the dividend delivered to the shareholders of the firm. In each period, individual productivity $\varphi$ is preserved in next period with probability $p$. If $\varphi$ is lost, with probability $1 - p$, a new productivity is drawn from a distribution function $F(\varphi)$. When $p = 1$, it is degenerate to the case with iid productivity shock. For tractability, we assume the productivity shock is independent from net worth. Therefore the joint distribution distribution $H(a, \varphi)$ is the product of $F(\varphi)$ and $G(a)$, the distribution of productivity and net worth respectively. Moreover, we assume the aggregate productivity affects production through its effect on $F(\varphi)$. Therefore we denote the productivity distribution as $F(\varphi, z)$. After all, it is the product of aggregate and individual productivity that matters for production.

**Credit Imperfections.** In complete market, heterogeneity in productivity incurs no efficiency loss since only the most productive entrepreneur will produce. As emphasized by Buera and Moll (2013), it is the interaction between productivity heterogeneity and financial friction that matters for capital misallocation. In our model incomplete market emerges due to credit frictions. More
specifically, due to limited enforceability, productive entrepreneurs cannot borrow as much as they want.

**Labor Search Frictions.** To model equilibrium unemployment, we assume the labor market is subject to search friction. \( \varphi \) units of consumption goods are realized only after one unit of capital used by entrepreneur-\((a, \varphi)\) is matched with one unit of labor. As standard in the labor search literature, the production function is the one-to-one Leontief way at the “atom” level. There is no information asymmetry. Entrepreneur-\((a, \varphi)\) could either borrow and produce with posting a public displayed wage contract \( w(\varphi) \) in sub-market \( \varphi \), or lend to other entrepreneurs in credit market\(^{10}\). Even though we assume away the cost of wage posting, not all entrepreneurs choose to produce in the end. Wage contract is publicly displayed and worker with identical labor self-selects into active sub-markets \( \varphi \in A \subseteq \Phi \). See Figure 2.1 for an illustration. If worker goes to sub-market \( \varphi \) and gets matched, she obtains wage \( w(\varphi) \).

![Figure 2.1: Wage Posting by Active Entrepreneurs](image)

**Consumption and Saving.** At the end of the day, the relationship between entrepreneur and worker is terminated. Workers make hands-to-mouth consumption from wage income if matched or unemployment compensation if unemployed\(^{11}\). The borrower entrepreneur gains capital revenue,
pays back to lender entrepreneurs, makes her own consumption and the rest is used as net worth for the next period.

**State Variables and Timing.** The idiosyncratic state variable is two dimensional, \((a, \varphi)\), the net worth and productivity. The aggregate state is denoted as \(X = (z, \lambda, \eta, H(a, \varphi))\), where \(z\) is the aggregate productivity shock, \(\lambda\) the credit efficiency, \(\eta\) the matching efficiency in labor market, and \(H(a, \varphi)\) the distribution on net worth and productivity. Given our assumption on productivity shock, the aggregate state can be rewritten from \(X = (z, \lambda, \eta, H(a, \varphi))\) to \(X = (\lambda, \eta, F(\varphi, z), G(a))\), where \(F(\varphi, z)\) and \(G(a)\) denotes the distribution of productivity and that of net worth respectively and the product yields their joint distribution. Finally, we present the time-line in Figure 2.2.

**Figure 2.2: Time-line**

2.2 Labor Market

We use competitive search with wage posting to model equilibrium unemployment. Similar to standard search theory, the matching function \(m(l(\varphi), r(\varphi))\) in any sub-market \(\varphi \in \Phi\) is assumed to be homogeneous of degree one, and increases with both arguments, where \(l(\varphi)\) and \(r(\varphi)\) denotes the measure of labor and capital in sub-market \(\varphi\) and the market tightness is denoted as \(\theta(\varphi) = \)
In turn, the matching probability of capital and that of labor, \( q(\theta(\varphi)) \) and \( p(\theta(\varphi)) \), has the following property: \( q' > 0, q'' < 0, p' < 0 \) and \( p'' > 0 \), where

\[
q(\theta(\varphi)) = \frac{m(l(\varphi), v(\varphi))}{v(\varphi)} = m(\theta(\varphi), 1) \\
p(\theta(\varphi)) = \frac{m(l(\varphi), v(\varphi))}{l(\varphi)} = m(1, \frac{1}{\theta(\varphi)}) = \frac{q(\theta(\varphi))}{\theta(\varphi)}.
\]

To sharpen our argument, we assume throughout this paper that the matching function in all submarket is assumed to be Cobb-Douglas, i.e., \( m(l(\varphi), v(\varphi)) = \eta \cdot l(\varphi)^{1-\gamma} \cdot v(\varphi)^\gamma \), where \( \eta \) denotes matching efficiency and is exogenously given throughout this paper. For simplicity, we remove the possibility of endogenous search effort in the baseline, but instead model firm’s search effort in Section 5. Moreover, we assume the cost of wage posting is zero in most part of the baseline model. Although each unit of labor is identical, due to search frictions and heterogeneity in capital productivity, there exists no unique and publicly displayed wage such that labor supply equals demand. Instead, we have the labor resource constraint as below.

\[
\int_{\Phi} l(\varphi) \cdot d\varphi = L. \tag{2.1}
\]

We formulate \( \pi(\varphi, W) \), the expected revenue of one unit of capital in submarket-\( \varphi \), as below.

\[
\pi(\varphi, W) \equiv \max_{\{\theta(\varphi, W), w(\varphi, W)\}} \left\{ q(\theta(\varphi, W)) \cdot (\varphi - w(\varphi, W)) \right\}, \tag{2.2}
\]

subject to

\[
p(\theta(\varphi, W)) \cdot w(\varphi, W) = W, \tag{2.3}
\]

where \( W(\varphi) = W(\varphi') \equiv W \) denotes the worker’s expected wage revenue by entering any submarket \( \varphi, \varphi' \in \Phi^A \subseteq \Phi \), where \( \Phi^A \) denotes the set of entrepreneurs active in production. Right now we treat \( \Phi^A \) as given. We use the following proposition to characterize the optimal wage scheme by active sub markets \( \varphi \in \Phi^A \).

**Proposition 1.** (Optimal Wage Scheme)

1. Given \( W \), the market tightness in active sub-markets \( \varphi \in \Phi_A \) is determined as below.

\[
q'(\theta(\varphi)) = \frac{W}{\varphi}, \tag{2.4}
\]
2. Wage scheme and expected capital revenue obtained from sub-market ϕ ∈ Φₐ is as follow.

\[ w(ϕ, W) = \frac{W}{p(θ(ϕ))} \]  \hspace{1cm} (2.5)

\[ π(ϕ, W) = Ξ(θ(ϕ, W)) \cdot ϕ, \]

where \( Ξ(θ) ≡ q(θ) − θq'(θ) \) and increases with \( θ \).

3. The comparative statics is given as below.

\[ \frac{∂π(ϕ, W)}{∂ϕ} > 0, \quad \frac{∂π(ϕ, W)}{∂W} < 0, \quad \frac{∂θ(ϕ, W)}{∂ϕ} > 0, \quad \frac{∂θ(ϕ, W)}{∂W} < 0, \quad \frac{∂q(θ(ϕ, W))}{∂ϕ} > 0, \quad \frac{∂q(θ(ϕ, W))}{∂W} < 0. \]

As shown in this proposition, rather than assuming a unique wage, the optimal wage contract implies the wage increases with the productivity of the corresponding entrepreneurs. In turn, the entrepreneurs with higher productivity has a higher probability to be matched with a worker.

2.3 Credit Market

In the baseline model, we address credit friction using collateral constraint as \( k ≤ λ \cdot a \) with \( λ ≥ 1 \), where \( k \) and \( a \) denotes the total capital available and own net worth respectively.\(^{14}\) If \( λ = 1 \), all entrepreneurs are in autarky. If \( λ = ∞ \), then there is no collateral constraint. Since entrepreneurs are heterogeneous in both net worth and productivity, we need to characterize under what conditions the collateral constraint is binding. Denote \( Π(k, ϕ) \) as the capital revenue of entrepreneur with \( k \) units of capital and productivity \( ϕ \). Based on the previous subsection, and assume law of large number applies, we know the capital revenue is linear with \( k \), i.e.,

\[ Π(k, ϕ) = π(ϕ, W) \cdot k, \]  \hspace{1cm} (2.6)

where \( π(ϕ, W) \) is characterized in Proposition 1. Then we reach the capital demand in Lemma 1.

**Lemma 1. (Capital Demand)** The capital demand by entrepreneur-(\( a, ϕ \)) ∈ \( A × Φ \) adopts either of corner solutions, i.e.,

\[ k(ϕ, a) = \begin{cases} λ \cdot a & \text{if } ϕ ≥ \hat{ϕ} \\ 0 & \text{if } ϕ < \hat{ϕ} \end{cases}, \]

where \( \hat{ϕ} \) is determined by

\[ π(\hat{ϕ}, W) = r \]  \hspace{1cm} (2.7)

and the set of active entrepreneurs is \( Φₐ = \{ ϕ | ϕ ≥ \hat{ϕ} \} \).

\(^{14}\)The assumption on homogeneous leverage ratio is purely for illustration simplicity. In Section 5, we will model an endogenous and heterogeneous leverage, which is also related to the endogenous interest rate.
Several comments are made. First, the above corollary immediately suggests the measure of capital in submarket \( \varphi \) is \( v(\varphi) = \lambda Af(\varphi) \cdot 1_{\{\varphi \geq \hat{\varphi}\}} \). Secondly, although \( k \in [0, \lambda \cdot a] \), the above lemma implies entrepreneurs adopt either of two corner solutions for capital demand. The property of choosing corner solutions is due to the linearity of capital gains. Only entrepreneurs with high enough productivity not only produce but also hit a binding collateral constraint. The rest prefer to lending in the credit market with interest rate \( r \). The clearing condition in credit market is then obtained as below.

\[
\int_{\Phi} \int_{A} k(\varphi, a) \cdot h(\varphi, a) d\varphi da = \int_{\Phi} \int_{A} a \cdot h(\varphi, a) d\varphi da. \tag{2.8}
\]

Moreover, using the lemma again with aggregate net worth denoting as \( A = \int_{A} a \cdot dG(a) \), the measure of capital in sub market \( \varphi \in \Phi \) is obtained as

\[
v(\varphi) = \left[ \int_{A} k(\varphi, a) dG(a) \right] \cdot f(\varphi) = \lambda Af(\varphi). \tag{2.9}
\]

Finally, we reach a corollary on double selection for production, which is also illustrated in Figure 2.3.

**Corollary 1. (Double Selection on Capital Use)**

1. The productivity distribution of capital engaging in production is obtained as below.

\[
F^A(\varphi) = \frac{F(\varphi) - F(\hat{\varphi})}{1 - F(\hat{\varphi})} < F(\varphi) \text{ for } \varphi \in \Phi.
\]

2. The productivity distribution of matched capital engaging in production is obtained as follows.

\[
F^M(\varphi) = \frac{\int_{\varphi}^{\hat{\varphi}} q(\varphi') \cdot dF(\varphi')}{\int_{\varphi}^{\varphi_{\text{max}}} q(\varphi') \cdot dF(\varphi')} < F^A(\varphi) \text{ for } \varphi \in \Phi.
\]

It is worth noting that the equilibrium productivity distribution of active entrepreneurs is \( F^M(\varphi) \) rather than \( F^A(\varphi) \). The latter is the truncated distribution in the first step. As proved in Proposition 1, the matching probability of active entrepreneurs increase their own productivity levels. As a result, the equilibrium productivity distribution is obtained after the second selection, which reflects in the weight \( q(\varphi) \) in the above equation on \( F^M(\varphi) \).

### 2.4 Entrepreneur’s Constrained Optimization

The constrained optimization of entrepreneur-\((a, \varphi) \in A \times \Phi \) is formulated as below.
Figure 2.3: Truncated Distribution of Productivity: The black solid line, the blue dotted line
and the green dashed line denote respectively the original distribution, the distribution of
active capital and the distribution of active capital that is finally matched with labor.

\[ V(a, \varphi; X) = \max \{ \log(d) + \beta \cdot \mathbb{E}[V(a', \varphi'; X') | X] \} \]  \hspace{1cm} (2.10)

subject to

\[ r \cdot b + d + i = \Pi(k, \varphi) \equiv \pi(\varphi) \cdot k \]  \hspace{1cm} (2.11)
\[ a' = (1 - \delta) \cdot a + i \]  \hspace{1cm} (2.12)
\[ b = k - a \]  \hspace{1cm} (2.13)
\[ k \leq \lambda \cdot a \]  \hspace{1cm} (2.14)
\[ k \geq 0. \]  \hspace{1cm} (2.15)

Equation (2.11) is the budget constraint with \( \Pi(k, \varphi) = \pi(\varphi) \cdot k \) being the capital income, \( r \cdot b \) the debt repayment, \( d \) the consumption and \( i \) the investment for next period. The corresponding balance sheet is shown in Table 1. Equation (3.3) is the accounting identity on investment, net worth and the total capital obtained for production. Equation (2.13) is the definition on external funding \( b \). Equation (2.14) is collateral constraint, in which the maximum available capital is proportional to entrepreneur’s own net worth. Finally, Equation (2.15) assumes no short sale. The collateral constraint \( k \leq \lambda a \) implies the leverage ratio is the same across heterogeneous entrepreneurs, and has nothing to do with interest rate. This is purely for tractability. We show in the appendix the collateral constraint which considers both interest rate and heterogeneity. It is still tractable at both micro and aggregate level. As emphasized by Moll (2012), it is the linearity of collateral constraint that guarantees tractability.
\[
\begin{array}{c|c|c|c|c}
\text{Assets} & \text{Liability} & \Rightarrow & \text{Assets} & \text{Liability} \\
\hline
a & a & \text{net worth} & k(\varphi, a) & a + b(\varphi, a) \\
\end{array}
\]

\[
\begin{align*}
\text{Assets} & \quad \text{Liability} \\
\text{Output} & = \Pi(k, \varphi) + W \cdot l(\varphi, a) & \text{Debt payment} & = [r + (1 - \delta)] \cdot b(\varphi, a) \\
\text{Depreciated capital} & = (1 - \delta) \cdot k(\varphi, a) & \text{Wage bill} & = W \cdot l(\varphi) \\
\text{New equity} & = (1 - \delta) \cdot a + i(a, \varphi) & \text{Depreciated equity} & = \Psi(\varphi) \cdot a \\
\end{align*}
\]

\[\text{Table 1: Evolution of Balance Sheet by Entrepreneur-(a, } \varphi)\]

\textbf{Lemma 2. (Policy Function)}

1. The constrained optimization by entrepreneur-(a, } \varphi)\) can be rewritten as

\[V(a, \varphi; X) = \max \{ \log(d) + \beta \cdot \mathbb{E}[V(a', \varphi'; X') | X] \},\]

subject to

\[c + a' = \Psi(\varphi) \cdot a,\]

where \(\Psi(\varphi) = \max \{ \pi(\varphi) - r, 0 \} \cdot \lambda + [r + (1 - \delta)]\).

2. The policy function by entrepreneur-(a, } \varphi)\) as below.

\[c(a, \varphi) = (1 - \beta) \cdot \Psi(\varphi) \cdot a\]

\[a'(a, \varphi) = \beta \cdot \Psi(\varphi) \cdot a.\]

Consequently, we obtain the firm size and the growth rate as below.

\textbf{Corollary 2. (Firm Size and Growth Rate)}

1. The firm size of entrepreneur-(a, } \varphi) \in A \times \Phi_A, \text{ measured by capital } k \text{ and employee numbers}
are obtained as below.

\[ k(a, \varphi) = \lambda \cdot a \]
\[ n(a, \varphi) = \lambda \cdot q(\varphi) \cdot a. \]

2. The growth rate of firm size is independent of firm size itself.

\[
\mathbb{E} \left[ \frac{k'}{k} | (k, \varphi; X) \right] = \beta \cdot \left( \frac{\Psi(\varphi, X)}{\lambda} \right) \cdot \mathbb{E} \left[ \mathbf{1}_{\{\varphi' \geq \hat{\varphi}(X')\}} \cdot \lambda'|(\varphi, X) \right]
\]
\[
\mathbb{E} \left[ \frac{n'}{n} | (n, \varphi; X) \right] = \beta \cdot \left( \frac{\Psi(\varphi, X)}{\lambda \cdot q(\varphi, X)} \right) \cdot \mathbb{E} \left[ \mathbf{1}_{\{\varphi' \geq \hat{\varphi}(X')\}} \cdot \lambda' \cdot q(\theta(\varphi', X')|(\varphi, X)) \right].
\]

3 Equilibrium Characterization

We use this section to launch general equilibrium analysis on both the stage equilibrium \((\hat{\varphi}, r, W)\), and then the transition dynamics on \(X = (\lambda, \eta, F(\varphi, z), A)\). We start by defining the recursive competitive equilibrium.

**Definition 1. (Recursive Competitive Equilibrium)** A recursive competitive equilibrium is (1) labor supply \(l(\varphi)\), capital \(v(\varphi)\) and market tightness \(\theta(\varphi)\) at active sub-market \(\varphi \in \Phi_A\), (2) a set of price functions, including the interest rate \(r\), the wage scheme \(w(\varphi)\) and the expected labor gain from sub-market \(W(\varphi)\) in active sub-market \(\varphi \in \Phi_A\), (3) a set of policy functions, including dividends \(d\), debt \(b\), and net worth for next period \(a'\), (4) the value function \(V(a, \varphi)\), and (6) the law of motion for the aggregate state-variable vector \(X = (z, \lambda, \eta, F(\varphi, z), A)\) such that, (i) given \(X\) and \(W\) the market tightness \(\theta(\varphi) = l(\varphi)/v(\varphi)\) is determined by Equation \((2.4)\), \(v(\varphi)\) is by Equation \((2.9)\) and wage \(w(\varphi)\) is solved by Equation \((2.5)\), (ii) given \(X\), the cut-off point, \(\hat{\varphi}\), the interest rate \(r\), and the expected wage revenue \(W\) are jointly determined by Equations \((2.7), (3.1)\), and \((3.2)\), (v) credit market clears (vi) \(d(a, X)\) and \(a'(a, X)\) is the solution to the entrepreneur’s dynamic optimization, and the value function \(V(a, X)\) is obtained with \(d(a, X)\) and \(a'(a, X)\).

3.1 Analytical Characterization

Using Lemma 1 helps simply the clearing condition as below.

\[
\lambda[1 - F(\hat{\varphi})] = 1
\]

Consequently, the labor resource constraint in Equation \((2.1)\) can be rewritten as

\[
\int_{\Phi_A} v(\varphi)\theta(\varphi) \cdot d\varphi = \lambda A \int_{\Phi_A} \theta(\varphi)dF(\varphi) = L.
\]
As implied by Equation (3.2), the aggregate state variable $X$ can be simplified as $X = (\lambda, \eta, F(\varphi, z), A)$. Since $(\lambda, \eta)$ is governed by some exogenous stochastic process, we reach the following proposition on characterizing $(\hat{\varphi}, r, W)$ and the transition dynamics on $F(\varphi, z)$ and $A$.

**Proposition 2. (General Equilibrium)**

1. In each period, given $X = (\lambda, \eta, F(\varphi, z), A)$, $(\hat{\varphi}, r, W)$ are jointly determined by Equations (2.7), (3.1), and (3.2).

2. The productivity distribution is recursively determined as below.
   
   $$F_{t+1}(\varphi) = p \cdot F_t(\varphi) + (1 - p) \cdot F(\varphi).$$

3. Given $X$ and denote $\overline{A} \equiv A/L$, the aggregate output and unemployment are characterized as below.
   
   $$Y = \left[ \int_{\Phi_A} \varphi \cdot q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda A$$
   $$u = 1 - \left[ \int_{\Phi_A} q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda \overline{A}.$$

4. The dynamics on aggregate net worth is governed by
   
   $$A_{t+1} = \beta \cdot \left[ \int_{\Phi} \Psi_t(\varphi) dF_t(\varphi) \right] \cdot A_t.$$

5. Given $F_t(\varphi)$ and $G_t(\varphi)$, $G_{t+1}(\varphi)$ is endogenously governed as below.
   
   $$G_{t+1}(a) = \int G_t \left( \frac{a}{\beta \cdot \Psi_t(\varphi)} \right) \cdot dF_t(\varphi)$$

Furthermore, we get sharper results by imposing some reasonable assumptions as below.

**Corollary 3. (Analytical Results)** If $\overline{F}(\varphi) = 1 - \left( \frac{\varphi}{\bar{z}} \right)^{-\frac{1}{\alpha}}$, and $F_0(\varphi) = 1 - \left( \frac{\varphi}{\bar{z}_0} \right)^{-\frac{1}{\alpha}}$, i.e., both $\overline{F}(\varphi)$ and $F_0(\varphi)$ are Pareto distributed, and the matching function in sub-market is $m(l(\varphi), v(\varphi)) = \eta \cdot l(\varphi)^{1-\gamma} v(\varphi)^\gamma$, then we have:

1. $F_{t+1}(\varphi)$ also conforms to Pareto distribution such that
   
   $$F_{t+1}(\varphi) = 1 - \left( \frac{\varphi}{z_{t+1}} \right)^{-\frac{1}{\alpha}}$$
   $$z_{t+1} = p \cdot z_t^{\frac{1}{\alpha}} + (1 - p) \cdot \bar{z}^{\frac{1}{\alpha}}.$$

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2. The truncated distribution on productivity is

\[ F^A(\varphi) = 1 - (\varphi/\hat{\varphi})^{-1/\alpha} \quad \text{for} \ \varphi \geq \hat{\varphi} \]

\[ F^M(\varphi) = 1 - (\varphi/\hat{\varphi})^{-1/\tilde{\alpha}} \quad \text{for} \ \varphi \geq \hat{\varphi} \]

where \( \tilde{\alpha} \equiv 1/(\frac{1}{\alpha} + 1 - \frac{1}{\gamma}) \) and thus \( \tilde{\alpha} > \alpha \).

3. The aggregate output \( Y \), the matched workers and unemployment is

\[ Y = \eta \hat{\varphi} \left( \frac{\gamma}{\gamma - \alpha} \right) \gamma A^\gamma L^{1-\gamma} = N \cdot E_{F_M}(\varphi) \]

\[ u = 1 - \eta \cdot \left[ \frac{\gamma^\gamma (\gamma - \alpha)^{1-\gamma}}{\gamma - \alpha + \gamma \alpha} \right] \cdot \left( \frac{A}{L} \right)^\gamma \text{ Beveridge Curve.} \]

4. \( (\hat{\varphi}, r, W, w(\varphi)) \) is obtained as below.

\[ \hat{\varphi} = \lambda^\alpha \cdot z \]

\[ r = \left( 1 - \frac{\alpha}{\gamma} \right) \cdot \left( \frac{\partial Y}{\partial A} \right) \]

\[ W = \frac{\partial Y}{\partial L} \]

\[ w(\varphi) = (1 - \gamma) \cdot \varphi \quad \text{for} \ \varphi \in \Phi_A = \{ \varphi | \varphi \geq \hat{\varphi} \} \]

5. The dynamics on aggregate net worth is governed by

\[ A_{t+1} = \beta \cdot [\gamma \cdot Y_t + (1 - \delta) \cdot A_t] \quad (3.3) \]

This corollary offers a micro-foundation for Cobb-Douglas aggregation with both financial friction and labor search friction. Our model is characterized with both equilibrium wage dispersion at the micro level and the tractability on aggregation and transition dynamics.

It is worth noting that, given the aggregate net worth \( A \) in current period, the unemployment \( u \) has nothing to do with financial friction \( \lambda \). Here is the intuition. When \( \lambda \) decreases, more unproductive entrepreneurs switch from lender to borrower for production. On the one hand, more jobs are created at the lower productivity level, which decreases unemployment. On the other hand, the amount of capital used by high productivity entrepreneurs decreases and thus some high level jobs are destructed. The net effect by the decrease of \( \lambda \) is in general ambiguous.

The above corollary suggests, when the maximum leverage ratio is the same across heterogeneous entrepreneurs, the reallocation effect by \( \lambda \), which consists of both job destruction and job creation, is zero in total. However, this is not not necessarily true in general case. As shown in
Section 5, given homogeneous leverage ratio, the introduction of firm’s endogenous search effort will deliver a outward shift of Beveridge curve when negative shock to \((z, \eta, \lambda)\) arrives. Alternatively, even in the absence of endogenous recruiting effort, when entrepreneurs face heterogeneous leverage, which makes sense since the collateral value of capital varies across entrepreneurs, then given \(A\), the decrease of \(\lambda\) delivers a decrease of unemployment too. That is, the job creation effect is dominant over the job creation effect with a credit crunch. However, the steady state unemployment decreases with \(\lambda\) since the aggregate net worth is positively related with \(\lambda\), and such credit channel dominates the reallocation channel.

[Insert A Figure to Illustrate the Reallocation Effect of \(\lambda\)]

We have so far set up and characterized a model with both capital and labor misallocation. The former misallocation is due to collateral constraint in credit market while the latter stems from search frictions in labor market. Using Corollary 3, we make a connection between our model with dual frictions and the model with only financial frictions.

**Corollary 4. (Isomorphism)** Our heterogeneous-entrepreneurs model with both search frictions in labor market with \(m(l(\varphi), v(\varphi)) = \eta \cdot l(\varphi)^{1-\gamma} v(\varphi)\), and financial frictions in credit market with \(k \leq \lambda \cdot a\) delivers the same output aggregation and transition dynamics on \(F(\varphi), G(a)\) and \(A\) with the model with the following characteristics:

1. The production function by entrepreneur-\((a, \varphi)\) is
   \[
y(\varphi, a) = \varphi \cdot m(k(\varphi, a), l(\varphi, a)).
   \]

2. The labor market is *frictionless* in each period, *i.e.*, there exists a unique and publicly displayed wage such that labor supply \((L)\) equals demand.

3. The credit market is subject to collateral constraint, *i.e.*, \(k \leq \lambda \cdot a\).

Moreover, we can use the model with only financial friction to recover the unemployment in the model with dual frictions as below.

\[
u = 1 - \frac{Y}{L \cdot E_{F_{M}}(\varphi)}.
\]

### 3.2 TFP and ALP

For each period, given the aggregate net worth \(A\) and labor supply \(L\), Corollary 3 provides a micro-foundation for the Cobb-Douglas aggregation. Moreover, we can define the TFP as

\[
\text{TFP} = \frac{Y}{A^\gamma L^{1-\gamma}} = \eta \left(\frac{\gamma}{\gamma - \alpha}\right)^\gamma \hat{\varphi} = z \cdot \eta \cdot \lambda^\alpha \cdot \left(\frac{\gamma}{\gamma - \alpha}\right)^\gamma.
\]
That is, the endogenous productivity efficiency is related to aggregate productivity shock \( z \), the matching efficiency in labor market \( \eta \), and the leverage ratio in credit market \( \lambda \). As a result, both labor search friction and financial friction work through the productivity efficiency at the macro level.

When calculating the TFP, as suggested by Lagos (2006), we can alternatively use the finally matched capital and labor, i.e., \( L_M = K_M = N \). That is, we have

\[
APL \equiv \frac{Y}{N} = \frac{Y}{A_M^\gamma L_M^{1-\gamma}} = \mathbb{E}_F^M(\varphi) = \left( \frac{\gamma - \alpha + \alpha \gamma}{\gamma - \alpha} \right) \cdot \lambda^\alpha \cdot z.
\]

Similar to the results on TFP, both \( z \) and \( \lambda \) contributes to ALP. However, ALP has nothing to do with \( \eta \). This is because the effect of matching efficiency is already integrated into the matched capital and labor.

### 3.3 Cost of Wage Posting

We have so far assumed away firm’s recruiting effort, which consists of two aspects. One is the cost of wage posting while the other one is endogenous search effort by entrepreneurs. We close this part with a short discussion on the cost of wage posting. We address the firm’s endogenous search effort in Section 5.

To activate one unit of vacancy, we now need \( \kappa \) units of capital. Since one vacancy corresponds to one capital, active entrepreneurs will divide every unit of their capital into \( (\frac{1}{1+\kappa}, \frac{\kappa}{1+\kappa}) \) for the vacancy and the cost of vacancy respectively. Everything proceeds very similarly to the baseline. In particular, the output and the unemployment is now modified as below.

\[
Y = z \eta \lambda^\alpha \left( \frac{\gamma}{\gamma - \alpha} \right)^\gamma \left( \frac{A}{1+\kappa} \right)^\gamma L^{1-\gamma}
\]

\[
u = 1 - \eta \cdot \left[ \frac{\gamma^\gamma (\gamma - \alpha)^{1-\gamma}}{\gamma - \alpha + \gamma \alpha} \right] \cdot \left[ \frac{A}{(1+\kappa)L} \right]^\gamma
\]

That is, the cost of wage posting imposes a tax wedge on capital use. It distorts downward TFP and increases unemployment, holding everything else unchanged. However, \( \kappa \) has nothing to do with \( \tilde{\varphi} \), the cut-off point for active production. Here is the intuition. When \( \kappa \) increases, both the demand and supply of capital decreases. Since entrepreneurs are assumed to have access to only intra-group lending, these two effects just cancel with each other. In turn, the cost of wage posting will not affect APL, which is proportional to \( \tilde{\varphi} \).

Consequently, \( \kappa \) and \( \eta \) delivers very similar implications at both aggregate and disaggregate levels. On one hand, both of them affects TFP and shifts the Beveridge curve. On the other hand, both of them has nothing to capital reallocation and generates the acyclicity of APL.
4 Quantitative Analysis

We use this section to check which shocks are in line with both aggregate and disaggregate empirical regularity laid down in the Introduction part. We first calibrate the model to US data. After that, we investigate the implication of candidate shocks at the macro level, say for output, unemployment and \( ALP \), and then at the micro level, say for the reallocation of capital use and the share of employment. We assume \( \kappa = 0 \) throughout this section.

4.1 Calibration

Denote \( r_e = \frac{1}{\beta_e} - (1 - \delta) \). In steady state, the evolution on aggregate net worth in Equation 2.12 implies \( \frac{A}{Y} = \frac{r_e}{\gamma} \), where

\[
A = \left( \frac{\eta z \gamma \lambda \alpha}{r_e} \right)^{\frac{1}{1 - \gamma}} \left( \frac{\gamma}{1 - \alpha} \right)^{\frac{\gamma}{1 - \gamma}} \cdot L
\]

\[
Y = \left( \frac{r_e}{\gamma} \right) \cdot A
\]

\[
\frac{D}{Y} = \left( \frac{D}{A} \right) \left( \frac{A}{Y} \right) = \left( 1 - \frac{1}{\lambda} \right) \left( \frac{\gamma}{r_e} \right)
\]

\[
u = 1 - \left[ \frac{\eta \gamma (\gamma - \alpha)^{1 - \gamma}}{\gamma - \alpha + \alpha \gamma} \right] \cdot \left( \frac{A}{Y} \right) \gamma.
\]

As a result, in the steady state, both \( A \) and \( Y \) increases with \( \lambda \) while \( u \) decreases with \( \lambda \). Most of the parameters are standard in the literature. We normalize \( z = 1 \). We take the depreciation rate as \( \delta = 0.06 \). We make \( \beta = 1/1.04 \) to let the risk-free interest rate as 4%. Following Petrosky-Nadeau (2011), we set \( \eta = 0.66 \). The elasticity parameter is respectively 0.5 in Petrongolo and Pissarides (2001) and 0.28 in Shimer (2005). Also notice that, since the optimal contract is \( w(\varphi) = (1 - \gamma)\varphi \), \( \gamma \) also denotes the share of capital revenue, which is typically 0.33 in US, which lies between the above two values. Thus we set \( \gamma = 0.33 \). The additional calibration in our model includes \( (\lambda, \alpha) \). It can be done by using the first moment on \( \left( \frac{D}{Y}, u \right) \). First of all, according to Beck, Demirguc-Kunt and Levine (2000), the external finance to GDP ratio is around 2.53 in US, which in turn suggests \( \lambda = 4.28 \). Finally, observe that \( u \) is a function of \( \alpha \). The long run unemployment rate in US is around 5.9%, which in turn suggests \( \alpha = 0.2967 \). We summarize the parameterization in Table 2.
4.2 Estimation and Impulse Response Exercise

As shown in Buera et al (2013), we use the recent financial crisis as an example. The aggregate output dropped by about 8% in 2008. As shown in previous section, both TFP and output is proportional to $\eta z \lambda^{\alpha}$. Thus we make the first drop as 8%, 8% and $8\% / \alpha$ respectively to produce the initial drop of 8% for output.

In general, both $u_t$ and $D_t/Y_t$ are functions of $(\lambda_t, \eta_t)$, which in turn helps us back out their values. This is our task in the near future. Here we just treat denote the auto-correlation for each shock as $\nu_t$. Then we make two pieces of comparison. One is on the impulse response to different shocks while the other one on the dynamic effect with different correlation. We document the impulse response exercise in Figure 4.1.

Several comments are worth made here. First, since the shocks to aggregate productivity, to matching efficiency in labor market and to the collateral constraint in credit market all work as productivity efficiency, they share the very similar impulse response to output, TFP, capital, unemployment, and capital return. Secondly, given the same initial shock to output, the shock to matching efficiency demonstrate a larger dynamic effect on unemployment and capital. Here is the intuition. Relative to $(\lambda, z)$ shock, the $\eta$ shock not only affects the evolution of aggregate net worth $A$, which in turn affects unemployment through the credit channel, but also directly works on unemployment by definition. Thirdly, it suggests the correlation matters. In any of these three panels, the higher the correlation is, the longer persistence and larger response to gap between unemployment and output recovery.

4.3 Which Shocks Really Matter?

A natural question is, which shock is most essential driving the jobless recovery? As suggested in Figure 4.1, despite difference in quantitative magnitude, the shocks to $(z, \eta, \lambda)$ deliver very similar implication for the dynamics of aggregate variables of main interest, such as output, TFP, capital,
Figure 4.1: Impulse response to aggregate productivity shock \((z)\), matching efficiency in labor market \((\eta)\) and collateral constraint in credit market \((\lambda)\).
unemployment and interest rate. Thus it is difficult, if possible, for us to identify the essential source for jobless recovery. Moreover, it is very likely that all these shocks work together in the past recessions.

However, if we switch from the aggregate to micro level implication, then shock in credit market $\lambda$ seems the most reasonable candidate. Here is the argument. The empirical finding by Kehrig reveals that, the productivity dispersion widens in recessions. Furthermore, he shows it is mainly due to the increasing proportion of unproductive firms. Through the lens of our model, the empirical regularity is that, $[1 - F(\hat{\phi})]$ is procyclical. As we can prove, the shock to either $z$ or $\eta$ does not change $[1 - F(\hat{\phi})]$ at all. However, the negative shock to $\lambda$ implies $[1 - F(\hat{\phi})]$ and the aggregate output are positively related.

5 Model Extension

We launch three pieces of model extension. First, we endogenize firm’s recruiting effort, which offers additional insight on the connection of financial market and labor market. The second extension is to model heterogeneous leverage, which is motivated by some recent empirical finding. Besides, we take into account the price effect, i.e., the role of interest rate in the leverage in both partial and general equilibrium. It then allows us to check whether there exists feedback from labor market to credit market. Finally, we introduce the financial intermediary into the model. We have so far assumed only an intra-group lending is available in our baseline model.

5.1 Endogenizing Firm’s Search Effort

As emphasized by Davis, Faberman and Haltiwanger (2012), in addition to posting vacancies, firm’s recruiting effort also include “increase advertising or search intensity per vacancy, screen applicants more quickly, relax hiring standards, improve working conditions, and offer more attractive compensation to prospective employees”. To this end, we follow We follow Pissarides (2000) to endogenize firm’s search effort\textsuperscript{15}

\textsuperscript{15}The recent work by Mukoyama, Patterson and Sahin (2013) complements to Davis, Faberman and Haltiwanger (2012) by focusing on the job search intensity of worker side. Our paper focuses on the firm size.
For tractability, we assume firm’s search effort is made after observing the aggregate state variable, but before the realization of their own productivity level. Moreover, we assume entrepreneurs use the worker’s labor input to increase search effort $s$, which may include advertising and screening effort. The labor input for firm’s recruiting effort is not related to that for production. More specifically, we assume $\sigma \in (0, 1)$ part of capital revenue is pledgeable to workers. Immediately, if $\sigma = 0$, then $s = 0$ and it is degenerate to the baseline. In each sub labor market $\varphi$, the matching function is $m(l(\varphi), v(\varphi)e(\bar{s}))$, where $\bar{s}$ denotes the average recruiting effort. Each entrepreneur treats $\bar{s}$ as given. In equilibrium, we have $s = \bar{s}$. Denote the market tightness as $\theta(\varphi) \equiv \frac{l(\varphi)}{v(\varphi)e(\bar{s})}$. We assume $e(0) = 1$. That is, when none of entrepreneurs exert search effort, then we are back to the baseline model. Moreover, we assume $c(0) = 0, c'(s) > 0$, and $c''(s) \geq 0$. In particular, we will specify $c(s) = \vartheta \cdot s$ for simplicity. Then the job filling rate and job finding rate is now adjusted as below, where $\bar{s}$ denotes the average search effort.

$$
q(\theta(\varphi), s) = \frac{m(l(\varphi), v(\varphi)e(\bar{s}))}{v(\varphi)e(\bar{s})} \cdot e(s) = m(\theta(\varphi), 1) \cdot e(s)
$$

$$
p(\theta(\varphi)) = \frac{m(l(\varphi), v(\varphi)e(\bar{s}))}{l(\varphi)} = m(1, \frac{1}{\theta(\varphi)}) q(\theta(\varphi), s) = \frac{\theta}{\theta(\varphi)} \cdot e(s).
$$

Then, given $(s, \bar{s})$, the decision by entrepreneur-$(a, \varphi)$ is formulated as below.

$$
\pi(\varphi, s) \equiv \max_{s.t. \ p(\theta(\varphi)) \cdot w(\varphi) = W} \{q(\theta(\varphi), s) \cdot (\varphi - w(\varphi))\}
$$

The FOC is $\frac{\partial q(\theta(\varphi), s)}{\partial \theta(\varphi)} = \frac{W \cdot e(s)}{\varphi}$, and therefore, as in the baseline, we have $\frac{\partial m(\theta(\varphi), 1)}{\partial \theta(\varphi)} = \frac{W}{\varphi}$. Then, given $(W, \varphi)$, the modified market tightness $\theta(\varphi)$ is pinned down. We summarize the key findings with endogenous search effort in the following corollary.

**Corollary 5.** Given the average search effort $\bar{s}$,

1. The cutoff point for production is $\bar{\varphi} = \lambda^\alpha z$, $ALP = \left(\frac{\gamma - \alpha + \gamma \alpha}{\gamma - \alpha}\right) \cdot \bar{\varphi}$ and the wage scheme is $w(\varphi) = (1 - \gamma)\varphi$, just as that in the baseline.

---

\footnote{We can alternatively assume entrepreneurs themselves incur non-pecuniary disutility to increase recruiting effort. Finally, we use the Appendix B to offer this alternative way to endogenize firm’s recruiting effort. It turns out these two approaches share very similar qualitative results.}

\footnote{$\sigma = 0$ is a sufficient condition for $s = 0$. Intuitively, there exists a cut-off point for $\sigma$ below which the corner solution on $s$ emerges.}

\footnote{That $c''(s) < 0$ is not sufficient to make the decision on search effort well defined. For the specification with $e(s) = (1 + s)^{\varepsilon}$, we must assume $\varepsilon \in (0, \frac{\alpha}{\alpha})$ rather than simply $\varepsilon \in (0, 1)$. This is used to make sure the expected revenue of expected revenue is concave in $s$.}
2. The aggregate output, unemployment is adjusted as below.

\[ Y = TFP \cdot A^\gamma \cdot L^{1-\gamma} \]

\[ u = 1 - \eta \cdot [e(\bar{s})]^{\gamma} \cdot \left[ \frac{\gamma^\gamma (\gamma - \alpha) 1^{1-\gamma}}{\gamma - \alpha + \gamma \alpha} \right] \cdot \left( \frac{A}{L} \right)^\gamma \]

where \( TFP = \eta \lambda^\alpha \cdot z \left( \frac{\gamma}{\gamma - \alpha} \right)^{\gamma} \cdot [e(\bar{s})]^{\gamma}. \)

3. The interest rate and expected wage revenue is

\[ r = \left( 1 - \frac{\alpha}{\gamma} \right) \cdot \left( \frac{\partial Y}{\partial A} \right) \text{ and } W = \frac{\partial Y}{\partial L}, \]

both of which increases with \((\lambda, \eta, z, \bar{s})\).

As a result, given \( \bar{s} \), everything is almost the same as the baseline. In particular, given \( A \), the net reallocation effect due to \( \lambda \) is still zero when \( \bar{s} \) is held constant. Given \( X \), it remains for us to pin down the endogenous search effort \( s \) and thus the average search effort \( \bar{s} \). Given \( s \), the individual decision rule on lending or borrowing depends on \( \bar{s}(s) \), where \( \pi(\bar{s}(s), s) = r \). Then \( s \) is chosen as below.

\[ \max \{ \sigma \cdot \{ \lambda [1 - F(\bar{s}(s))] \cdot E[(\pi(\varphi, s) - r)|\varphi \geq \bar{s}(s)] + r \} \} - c(s), \]

where \( \lambda [1 - F(\bar{s}(s))] \cdot E[(\pi(\varphi, s) - r)|\varphi \geq \bar{s}(s)] + r \) denotes the expected capital revenue with search effort \( s \) by workers, and \( \sigma \) of it can be pledgeable to them, and \( c(s) \) denotes the effort cost. The above problem is simplified as

\[ \max_{s \geq 0} \left\{ (1 - \sigma) \lambda \int_{\bar{s}(s)}^{\varphi_{\text{max}}} \pi(\varphi, s) - r \cdot dF(\varphi) - c(s) \right\}. \]

We fully characterize the equilibrium \((s, \bar{s})\) in the following proposition.

**Proposition 3. (Endogenous Search Effort and Shift of Beveridge Curve)**

1. Given \( X \) and \( \bar{s} \), the decision on search effort \( s \) can be simplified as below.

\[ \max_{s \geq 0} \left\{ r(1 - \sigma) \left( \frac{\alpha}{\gamma - \alpha} \right) \cdot [1 - F(\bar{s}(s))] - c(s) \right\}, \]

where

\[ r(1 - \sigma) \left( \frac{\alpha}{\gamma - \alpha} \right) \cdot [1 - F(\bar{s}(s))] = \Delta \left( \frac{A}{L} \right)^{\gamma - 1} \lambda^\alpha \cdot \eta \cdot [e(\bar{s})]^{\gamma(1 - \frac{\alpha}{\gamma})} e(s) \bar{s}, \]

which is strictly concave with \( s \). \( \Delta \) is a constant related to \((\alpha, \gamma)\).

2. The optimal individual effort, \( s \), is characterized by the FOC as

\[ -r \lambda \sigma \left( \frac{\gamma}{\gamma - \alpha} \right) f(\bar{s}(s)) \left( \frac{d\bar{s}(s)}{ds} \right) \leq c'(s), \]

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where the equality holds when \( s > 0 \).

3. Given \( X \), the individual effort equals the average effort in equilibrium, \( i.e., \ s = \overline{s} \), and it is solved by the above equation with both \( r \) and \( W \) being a function of \( \overline{s} \). That is, denote \( \Delta' = (\frac{z}{\lambda}) \Delta \), we have
   \[
   s = \overline{s} = \left[ \left( \frac{\Delta'}{\phi} \right) \left( \frac{A'}{L} \right) \gamma^{-1} \lambda^\alpha \right]^{\gamma-1} \lambda \phi \eta.
   \]
   Thus \((s, \overline{s})\) increases with \( z, \eta, \lambda \) while decreases with \( \frac{A'}{L} \). In turn, the Beveridge curve shifts with these three shocks in equilibrium.

Several comments are made for this proposition. First, the equilibrium search effort \( \overline{s} \) increases with \((z, \eta, \lambda)\). That is, these shocks will be amplified through the search effort. Secondly, given the aggregate net worth \( A \), if search effort is held constant, the net effect from capital and labor reallocation is zero. However, the negative shock on aggregate productivity and collateral constraint will dampen search effort, which makes the job creation dominated by job destruction. Consequently, the unemployment increases and the Beveridge curve shifts outward. Thirdly, different from \((z, \lambda)\), the matching efficiency in the labor market, \( \eta \), does not affect unemployment through the reallocation channel. Instead, on one hand, as \((z, \eta)\) does, the decrease of \( \eta \) discourages equilibrium search effort and thus shifts outward the Beveridge curve. On the other hand, holding everything else unchanged, \( \eta \) directly affects unemployment by definition. These two effects work in the same direction. Fourthly, in partial equilibrium, the increase of \( \pi \) exerts negatively externality on the expected capital revenue of individual entrepreneurs. Thus the optimal level effort should be smaller than the one solved above. We can easily check that the optimal search effort still increases with \((z, \eta, \lambda)\) while decreases with \( \frac{A'}{L} \).

We close this part by characterizing the policy function by entrepreneur-\((a, \varphi)\) and the evolution of aggregate net worth is obtained as below.

\[
\begin{align*}
d(a, \varphi) &= \beta \cdot \Psi(\varphi, \overline{s}) \cdot a \\
\alpha'(a, \varphi) &= (1 - \beta) \cdot \Psi(\varphi, \overline{s}) \cdot a \\
A' &= \beta [\gamma(1 - \sigma)Y + (1 - \delta)A].
\end{align*}
\]

where \( \Psi(\varphi, \overline{s}) = (1 - \sigma) \cdot \{max (\pi(\varphi, \overline{s}) - r, 0) \cdot \lambda + r \} + (1 - \delta). \)

5.2 Heterogeneous Leverage

We have so far specified the credit friction as \( k \leq \lambda \cdot a \), a much simplified collateral constraint of Kiyotaki and Moore (1997), where \( \lambda \equiv 1 + \rho \) and \( \rho \in (0,1) \). That is, the maximum leverage
ratio is the same across heterogeneous entrepreneurs and has nothing to do with either interest rate or labor market conditions. This is purely for simplicity. As shown in the empirical study by Macnamara (2012), the cross-section distribution of firm leverage is positively skewed.

We use this part to show the general form in which leverage is related to interest rate as well as to entrepreneur’s individual productivity. A natural starting point is that we specify an ex post collateral constraint, i.e., \( r \cdot b \leq \rho \cdot \Pi(k, \varphi) \). Then the leverage ratio is \( \lambda(\varphi) = 1/[1 - (\rho/r) \cdot \pi(\varphi)] \), which increases with \( \rho \) and \( \pi(\varphi) \) while decreases with \( r \). To make the leverage ratio well defined, we have to assume \( \pi(\varphi) < r/\rho \) holds for \( \varphi \). The potential problem is, if \( \varphi_{max} = \infty \), which is true for Pareto distribution, then \( \pi(\varphi) < r/\rho \) will absolutely be violated for high enough \( \varphi \). To address this problem while still preserve tractability and leverage heterogeneity, we may consider an ex ante collateral constraint, i.e., the collateral constraint goes as \( r \cdot b \leq \rho \cdot \Pi(a, \varphi) \), which in turn suggests

\[
\lambda(\varphi) = 1 + \left( \frac{\rho}{r} \right) \cdot \pi(\varphi). \tag{5.1}
\]

Notice that the modified leverage \( \lambda(\varphi) \), similar to the original one, increases with \( \rho \) and \( \varphi \) and decreases with \( r \). Moreover, it is well defined even for large enough \( \varphi \). Thus we use this version of collateral constraint to model heterogeneous leverage. In turn, the capital demand by entrepreneur-\((\varphi, a) \in A \times \Phi \) is adjusted as

\[
k(\varphi, a) = \begin{cases} 
\lambda(\varphi) \cdot a & \text{if } \varphi \geq \hat{\varphi} \\
0 & \text{if } \varphi < \hat{\varphi}
\end{cases},
\]

where \( \hat{\varphi} \) is determined by \( \pi(\hat{\varphi}, W) = r \). Correspondingly, the measure of capital in sub-market \( \varphi \) is modified as \( v(\varphi) = A \lambda(\varphi) f(\varphi) \), the constraint on labor resource is characterized as below.

\[
\int_{\Phi_A} \theta(\varphi) \cdot \lambda(\varphi) \cdot dF(\varphi) = L/A, \tag{5.2}
\]

Moreover, the clearing condition in credit market is formulated as

\[
\int_{\Phi_A} \lambda(\varphi) \cdot dF(\varphi) = 1. \tag{5.3}
\]

Using the similar procedure in Proposition 2, we can obtain \((\hat{\varphi}, r, W)\) and the transition dynamics. We summarize the key result in the following corollary. Furthermore, we stick to the assumption that \( F(\varphi) \) is Pareto and the matching function is Cobb-Douglas.

**Corollary 6. (General Equilibrium with Heterogeneous Leverage)**

1. \((\hat{\varphi}, r, W)\) are jointly determined by Equations \([2.7], [5.2]\) and \([5.3]\). More specifically, we
have
\[ \hat{\phi} = \left[ 1 + \rho \cdot \left( \frac{\gamma}{\gamma - \alpha} \right) \right]^\alpha \cdot z \]
\[ r = \left( 1 - \frac{\alpha}{\gamma} \right) \left( \frac{\partial Y}{\partial A} \right) \]
\[ W = \frac{\partial Y}{\partial L}. \]

2. (Non-degenerate reallocation effect of credit friction on unemployment) The output, unemployment, and \( APL \) is obtained as below.

\[ Y = TFP \cdot A^\gamma L^{1-\gamma} \]
\[ u = 1 - \eta \cdot \left[ \frac{\left( \frac{\gamma}{\gamma - \alpha(1-\gamma)} \right) + \left( \frac{\gamma}{\gamma - \alpha(2-\gamma)} \right) \cdot \rho}{\left( \frac{\gamma}{\gamma - \alpha(1-\gamma)} \right) + \left( \frac{\gamma}{\gamma - \alpha(2-\gamma)} \right) \cdot \rho} \right] \cdot \left( \frac{A}{L} \right)^\gamma \]

where
\[ TFP = \eta \hat{\phi} \left[ \frac{\left( \frac{\gamma}{\gamma - \alpha} \right) + \rho \left( \frac{\gamma}{\gamma - 2\alpha} \right)}{1 + \rho \left( \frac{\gamma}{\gamma - \alpha} \right)} \right]^\gamma = \eta \left[ 1 + \rho \left( \frac{\gamma}{\gamma - \alpha} \right) \right]^{\alpha-\gamma} \left[ \left( \frac{\gamma}{\gamma - \alpha} \right) + \rho \left( \frac{\gamma}{\gamma - 2\alpha} \right) \right]^\gamma, \]

and thus \((Y, u, TFP)\) increases with \( \rho \).

3. The evolution on \( F(\phi), G(a) \) and \( A \) are the same as in the baseline, with \( \Psi(\phi) \) being modified as below.

\[ \Psi(\phi) = \max \left\{ \rho \left( \frac{\phi}{\phi} \right)^{\frac{2}{z}} + (1 - \rho) \left( \frac{\phi}{\phi} \right)^{\frac{2}{z}} - 1, 0 \right\} \cdot r + r + (1 - \delta) \]

Similar to the baseline, both the stage equilibrium and transition dynamics are highly tractable. Moreover, \( TFP \) is also clearly related with aggregate productivity \( z \), matching efficiency in the labor market \( \eta \), and the collateral constraint in the credit market \( \rho \equiv \lambda - 1 \). However, contrary to the baseline, given \( A \), the unemployment rate \( u \) increases with \( \lambda \). That is, even in the absence of endogenous recruiting effort by the firms, the credit shock can still deliver a shift of Beveridge curve if heterogeneous leverage works as in this part.

5.3 Financial Intermediary Capital

Since we assume workers make hand-to-mouth consumption, and the financial intermediary itself has no net worth, entrepreneurs only have access to intra-group borrowing and lending. To preserve tractability while amplifying the effect of financial friction on unemployment, we introduce financial
intermediary (FI) in the following way. Like workers, FI does not save, and has no knowledge on production. However, FI can transform her labor into capital goods with a one-to-one linear technology. The objective function is given as below.

\[ U_f = \mathbb{E} \left\{ \sum_{t=0}^{\infty} (\beta_f)^t \cdot \left[ -B_t + (r_{t-1} + 1 - \delta) \cdot B_{t-1} \right] \right\}, \]

where \( \beta_f \) denotes her discount factor, \( \delta \) the depreciation rate of capital and \( B_t \geq 0 \) the amount of capital goods produced by FI at the beginning of \( t \). It is then distributed to entrepreneurs, who pay back with consumption goods at the end of \( t \). We assume FI can only consume the goods at the beginning of \( t+1 \). In a nutshell, FI serves two roles, one is to intermediate between borrower and lender entrepreneurs. Direct lending is removed. We assume only FI can intermediate. The other one is to contribute her own net worth to entrepreneurs. Finally, we assume entrepreneurs are less patient than financial intermediary.

**Assumption 1.** \( 0 < \beta \leq \beta_f < 1 \).

The decision on supply of capital goods by fully competitive financial intermediary is characterized as below.

\[ \left[ -\frac{1}{\beta_f} + r + (1 - \delta) \right] \cdot B = 0, \quad -\frac{1}{\beta_f} + r + (1 - \delta) \leq 0, \quad B \geq 0, \quad (5.4) \]

where \( B \geq 0 \) denotes the amounts of capital goods produced by financial intermediary. Then the clearing condition in the credit market is adjusted as below.

\[ A \cdot \lambda \cdot [1 - F(\hat{\varphi})] = A + B \quad (5.5) \]

We summarize the key findings in the following proposition.

**Proposition 4.** In each period, given \( X \), there exists a cut-off point \( \hat{\lambda}(X) \) such that,

1. When \( \lambda \leq \hat{\lambda}(X) \), \( B = 0 \) and \((\hat{\varphi}, r, W)\) are the same as that in the baseline model.

2. When \( \lambda > \hat{\lambda}(X) \), \((r, \hat{\varphi}, W, B)\) is characterized as below.

\[ \hat{\varphi} = \left[ \left( \frac{r}{\gamma \eta} \right)^{\frac{1}{1-\gamma}} \left( \frac{\gamma}{\gamma - \alpha} \right)^{\frac{1}{\gamma}} \right]^{\frac{1}{\gamma}}, \]

\[ r = r_f \equiv \frac{1}{\beta_f} - (1 - \delta), \]

\[ W = \left[ \eta^{\frac{1}{\gamma}} \gamma (1 - \gamma)^{\frac{1}{\gamma}} \hat{\varphi}^{\frac{1}{\gamma}} / r \right]^{\frac{1}{\gamma}}, \]

\[ B = \{ \lambda [1 - F(\hat{\varphi})] - 1 \} \cdot A > 0. \]
3. Denote the aggregate capital as $K = A + B$. Then the aggregate output and unemployment is

$$Y = \eta \left( \frac{\gamma}{\gamma - \alpha} \right)^{\gamma} \hat{\varphi} \cdot K^{\gamma} L^{1-\gamma} = N \cdot E_{FM}(\varphi)$$

$$u = 1 - \frac{N}{L}.$$

The additional insight by introducing financial intermediary capital is intuitive. First of all, if $r_f$ is large enough, i.e., the cost of getting credit outside of the entrepreneurs as a whole is high enough, everything goes back to our baseline model. On the other hand, given $r_f$, when the leverage ratio is high enough, i.e., the credit market is developed highly enough, then the financial intermediary not only serves as credit intermediation, but also is willing to offer additional capital for productive entrepreneurs. In the latter scenario, the effect of financial friction has both reallocation effect and credit effect in each cross section. Thus the implication of credit friction is amplified when the bank has access to capital outside the entrepreneurs as a whole.

6 Conclusion

To explain jobless recovery associated with the past three recessions, we developed a heterogeneous-entrepreneurs model with two layers of frictions. One is credit friction through collateral constraint while the other is competitive search used to model equilibrium unemployment. At the micro level, entrepreneurs choose to be as borrowers or lenders according to their idiosyncratic productivity. Moreover, the optimal wage scheme by active entrepreneurs predicts an equilibrium wage dispersion for homogeneous workers. At the aggregate level, the output admits an endogenous Cobb-Douglas production under some reasonable assumptions. Besides, the TFP is shown to be neatly related to three sources of aggregate shocks: productivity, collateral constraint in credit market and matching efficiency in labor market.

Since all the aforementioned shocks work as productivity wedge at the aggregate level in equilibrium, they produce a similar impulse response for aggregate variables of interest, including output, unemployment, TFP, capital and interest rate. Moreover, our quantitative analysis reveals that the persistence of shocks really matter to support jobless recovery. In particular, the more persistent of certain shock, the more gap between output and employment gap and the more significantly the unemployment responses to the shocks. Therefore all these shocks may play an important role in the past recessions.

Our model with rich heterogeneity at the micro level further helps us to detect which shock
is most essential. The empirical regularity at the micro level suggests the productivity dispersion widens and the proportion of unproductive entrepreneurs increases in recessions. We use this fact to discipline our analysis. We prove that the proportion of unproductive entrepreneurs is unchanged with neither aggregate productivity shock nor the shock to matching efficiency in labor market. It is the shock of collateral constraint in credit market that generates a negative relationship between output and the proportion of unproductive entrepreneurs. Thus we claim it is the persistent shock on credit friction that matters not only for explaining jobless recovery, but also coincides with the micro-level empirical facts.

References


Appendix A - Proofs

In progress. Coming soon.

Appendix B - Alternative Way to Endogenize Recruiting Effort

In progress. Coming soon.