Endogenous Vertical Industrial Structure in Successive Oligopolistic Entry Game*

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3/15/2011

Abstract

This paper employs a game-theoretic model to analyze the incentives for firms to integrate vertically in a successive oligopolistic industry. Allowing an integrated firm to trade in the intermediate-good market, we find that vertical integration (separation) tend to arise when there are more upstream (downstream) firms from the outset. When the numbers of upstream and downstream firms are “close”, both maximum number of integrations and no integration at all can be Nash equilibrium outcomes. We also identify conditions for “partial vertical integration” where integrated and non-integrated firms coexist in a Nash equilibrium. In contrast with the political economy perspectives, our results provide strategic explanations for the determination of industrial structure in a vertically related market.

1 Introduction

The decision of vertical integration and outsourcing has taken center stage in economic analysis of high technology industries. Recently, ASUSTeK, a Taiwanese brand name computer manufacturer, separated its brand and ODM operation and formed a new company called Pegatron to specialize in design and manufacturing services. While this kind of “vertical separation” is quite common in the Taiwanese electronic industry, the structure of electronic industry in South Korea appears to be much more concentrated than that in Taiwan.

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*This research is supported by Center for Research in Economics and Strategy (CRES), in the Olin Business School, Washington University in St. Louis. The author is grateful to Amitay Alter, Gaetano Antinolfi, Sebastian Galiani, Hülya Karaman, Shintaro Miura, Juan Pantano, Yang Tang, Ping Wang, and Russell Wong for helpful advice and suggestions. Needless to say, any remaining errors are our sole responsibility.

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What are the reasons behind such a contrasting difference in the industrial structure in these two countries? One strand of literature studying this topic has adopted the political economy perspectives (Haggard, 1990; Mody, 1990; Whitley, 1992; Hobday, 1995; Fields, 1995; Weiss and Hobson, 1995; Mathews and Cho, 2000; Thurbon, 2001). These studies all focus on different institutional configuration for advancing economic catch-up, as well as on how those two countries differ in terms of managing their respective financial and industrial systems. Few studies, however, have tackled the issue of how both economies have been transitioning from catch-up-based to innovation-based economies, especially in the post-1997 financial crisis era.

Partly based on the findings of the above studies, this paper emphasizes the strategic interactions among firms in the high technology industry with a view to take the comparison one step further. Salinger (1988) is one of the earliest studies adopting a game-theoretic framework to analyze the vertically related industries. He finds that a vertical merger may result in market foreclosure and lower social welfare. This result, however, depends on his assumption that a vertically integrated firm will not trade in the intermediate-good market. Wang et al. (2005) relaxes this assumption and allow the vertically integrated firms to buy or sell intermediate goods. They find that a vertical merger never results in market foreclosure and never reduces social welfare in this setup.

While both of the above-mentioned papers treat the number of vertically integrated firms as exogenous, we make the vertical merger decision endogenous in our model because of our interest in knowing how the initial industrial configuration affects the firms’ incentives to integrate. To do so, we set up a three-stage game: Vertical integration in the first stage, followed by an upstream Cournot competition in the second and a downstream Cournot competition in the third. In the first stage, the existing firms in the downstream and upstream simultaneously decide whether to vertically integrate with one of its counterparts or not. In the second stage, the vertically integrated firm(s) and the independent upstream firm(s) compete in quantity in the intermediate-good market. Depending on the industrial configuration, an integrated firm might want to sell and make profits in the intermediate-good market or to strategically buy in that market in order to raise the costs of its opponents in the final-good market as suggested by Salop and Scheffman (1983). In the third stage, after the first and second stage decisions are made, the integrated firm(s) and the independent downstream firm(s) compete in quantity in the final good market.

We find that vertical integration (separation) tend to arise when there are more upstream (downstream) firms from the outset. Intuitively speaking, a vertical integration hurts the upstream firm and benefits the downstream firm. Therefore, one pair of firms chooses to merge vertically only when the downstream gains could more than offset the upstream losses. When there are many upstream firms, intensive competition reduces
the profit margin and lowers the opportunity cost for an upstream firm to integrate vertically. One the other hand, vertical integration enables a downstream firm to acquire the intermediate good at cost and facilitate it an option to raise its opponents costs through strategic buying in the intermediate-good market. While the gains from reducing production cost dwindles in the presence of many upstream firms, an integrated firm can take advantage of strategic buying and benefit substantially from increasing sales in the final-good market.

Due to the complex forces at work, when the numbers of upstream and downstream firms are “close”, both maximum number of integrations and no integration at all can be Nash equilibrium outcomes. We also find “partial vertical integration” where integrated and non-integrated firms coexist in equilibrium for some initial market configuration. Our model provides strategic explanations for the determination of industrial structure. In this sense, we suggest that the initial numbers of firms in upstream and downstream markets matter for the resulted market structures.

The paper is organized as follows. The next section introduces the basic concepts and gives first results. In sections 3 various factors that have an effect on firms’ incentives to vertical integrate are discussed, i.e., double marginalization problem and market foreclosure, and setup of market structure. The equilibrium in the production stage will be derived. Section 4 analyzes different equilibria for firms’ vertical integration decisions, and section 5 concludes.

2 The Model

In this section, the vertical integration decision in the IT industry is modeled as a three stage game: a vertical integration in the first stage, followed by upstream production stage in the second and the downstream production competition in the third stage. In the first stage, the existing firms in the downstream and upstream simultaneously decide whether to vertically integrate or not. The firms make their integration decision based on the anticipated equilibrium profits in the following production stages. Then we assume there is a Cournot oligopoly at each of two successive stages of production. In the second stage, the upstream firms decide how much to buy or to sell to the downstream market, taking as given the industry structure formed after the decision made in first stage. At last stage, on knowing the price of intermediary goods, the downstream firms make their production decisions by selling the final goods to the end consumers.

The timeline of the game is shown in the following flow chart.

We proceed to derive the subgame perfect equilibria of the integration game by making the following assumptions first.

- Given a vertically related industry structure, \( n_d \geq 2 \) firms compete in down-
stream final goods market and $n_u \geq 2$ firms compete in upstream intermediary good market.

- Both upstream and downstream firms compete in the Cournot oligopoly.

- Downstream firms produce by transferring one unit of intermediary good into one unit of final product.

- We normalize the downstream production cost to be zero and downstream firms have to pay a price $w$ for each unit of intermediary good used. Then each downstream firm sell each unit of final good in $p_d$ to the market.

- Upstream firms produce by using a constant return to scale technology. Therefore, the average unit cost for upstream firms can be represented as:

$$c_i = t$$

- Here $i$ is an index of upstream firms.

$$i \in \{1, 2, \ldots, m\}, \text{ this upstream firm is integrated with a downstream firm.}$$

$$i \in \{m + 1, m + 2, \ldots, n_u\}, \text{ this upstream firm is not integrated with a downstream firm.}$$

- A linear demand for final product is given by $p_d = a - Q$, where $Q = \sum_{i=1}^{n_d} q_i$ is the total final product sold and $q_i$ is the individual output in the downstream market.$^1$

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$^1$The demand side of the downstream market is a simplified version of Singh and Vives (1984). The inverse demand here is generated by the utility function: $U(q, m) = a \sum_{i=1}^{n} q_i - \frac{1}{2}(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j} q_i q_j) + m$, where $m$ is a numeraire good.
3 The equilibrium in the downstream and upstream production stage

Then we follow by solving the downstream Cournot oligopoly game first. Each downstream firm maximizes profit by choosing the quantities to produce.

\[
\max_{q_i} \pi_i = (a - Q - w)q_i = (a - \sum_{j=1}^{n_d} q_j - w)q_i
\]

Simply by solving standard first order conditions, the equilibrium in this stage is that each downstream firm produces

\[
q_i = \frac{1}{1+n_d}(a - w)
\]

The total final products \(Q = \frac{n_d}{1+n_d}(a - w)\) are sold in the market.

We now turn to the upstream stage. Denote the total upstream production \(Q_u = \sum_{i=1}^{n_u} q_{ui}\). Here \(q_{ui}\) is the production of each upstream company \(i\). Because the downstream company transfers one unit of intermediary product into a final product, we know the relation \(Q_u = Q = \frac{n_d}{n_d+1}(a - w)\) should hold. Then upstream firms face the inverse demand \(w = a - \frac{n_d+1}{n_d}Q_u\) in this stage. Thus, each upstream firm maximizes his profits by making the production decision.

\[
\max_{q_{ui}} \pi_{ui} = [a - \frac{n_d+1}{n_d}Q_u - t]q_{ui}
\]

Since the upstream firms also compete in quantity and the production decision can be easily derived by solving the standard first order condition. The only equilibrium is for each firm to produce \(q_{ui} = \frac{n_d}{(1+n_d)(1+n_u)}(a - t)\) Hence, in equilibrium,

\[
p_d = \frac{1 + n_u}{(1 + n_d)(1 + n_u)}a + \frac{n_d n_u}{(1 + n_d)(1 + n_u)}t
\]

\[
w = \frac{1}{1+n_u}(a + nu t)
\]

\[
Q = \frac{n_d n_u}{(1 + n_d)(1 + n_u)}(a - t) = Q_u
\]

Then we are to solve the vertical integration decision between upstream and downstream firms. However, now several cases are left to discuss. Assume first \(m \leq \min\{n_u, n_d\}\) to be the number of vertically integrated firms. In the one extreme, we may have \(m = 0\). This is the case that no vertically integrated firms exist in the market. In the other extreme, the maximum number of integrated companies occur in the market. We will then call the first case full vertical separation and the second case full vertical integration. The full vertical separation case has just been discussed. Now we allow the maximum number of vertically integrated firms \(m = \min\{n_u, n_d\}\), in
which the full vertical integration emerges. By the possible arising market structure, we may have either \( m = n_d \leq n_u \) or \( m = n_u < n_d \). We must discuss these two cases separately.

**Case One :** \( n_d \leq n_u \)

Whenever full vertical integration emerges in this case, there will be no demand for intermediary goods produced from independent upstream firms. Those vertically integrated firms will source their input demand from their own upstream departments since firms only have to pay marginal cost by sourcing inside. The problem is greatly simplified into an \( n_d \)-firm Cournot quantity competition in the downstream final good market. Each one of the \( n_d \) oligopolists maximizes his profits by making the production decision.

\[
\pi_i = \max_{q_i} \left[ a - \sum_{j=1}^{n_d} q_j - c_i q_i \right] \\
= \max_{q_i} \left[ a - \sum_{j=1}^{n_d} q_j - t q_i \right]
\]

The only equilibrium is symmetric and each firm produces \( q_i = \frac{1}{1+n_d}(a-t), \forall i \in [1,n_d] \).

For the decision of vertical integration, we have to compare the joint profits of separated vertical and downstream firms together and the profits of the vertically integrated firms. Thus we denote \( \hat{\pi}(m, n_d, n_u) \) as the equilibrium profits of an integrated firm and \( \pi^{vs}_d(m, n_d, n_u) + \pi^{vs}_u(m, n_d, n_u) \) as the equilibrium joint profits of an upstream and a downstream vertically separated firms. Then from the results solved above,

\[
\hat{\pi}(n_d, n_d, n_u) = \frac{(a-t)^2}{(1+n_d)^2} \\
\pi^{vs}_d(0, n_d, n_u) + \pi^{vs}_u(0, n_d, n_u) = \frac{(a-t)^2(n_u^2 + n_d^2 + n_d)}{(1+n_u)^2(1+n_d)^2}
\]

**Case Two :** \( n_u < n_d \)

In this case, even if under vertical integration, the integrated firms may still continue supplying the downstream competing firms. The vertically integrated firms are maximizing the joint profits of selling intermediary goods and final products. Thus we would analyze the partial integration in the market first that is \( m \in \{1,2,\ldots,n_u\} \) firms are integrated and then the full integration can be predicted as a limiting case when \( m \) approaches \( n_u \).

The partial integration can be described as \( m \in \{1,2,\ldots,\min\{n_u, n_d-1\}\} \), so that at least one downstream firm is not integrated. In order to ease the complication of analysis, we split the firms into vertically integrated firms indexed as \( i \in \{1,2,\ldots,m\} \) and non-integrated firms indexed as \( j \in \{m+1, m+2,\ldots,n_d\} \) in downstream market and \( j \in \{m+1, m+2,\ldots,n_u\} \) in upstream market. Here the game structure
is similar to what we discuss in the full vertical separation. In the final good production stage, the integrated firms and non integrated firms simultaneously determine the quantities \( q_i \) and \( q_j \) of final good production. Then in the preceding stage, the non integrated upstream firms and the integrated firms also compete in quantities bearing in mind the derived demand generated from the production decision of the final good stage. We denote the decision variable of the non integrated upstream firms as \( q_{uj} \). The vertical integrated firms care about his net sales to non integrated downstream firms in the upstream department which is denoted as \( s_i \).

In the final product stage, each firm maximizes its related profit function.

\[
\max_{q_i} (a - Q - t)q_i + (w - t)s_i, \forall i \in \{1, 2, \ldots, m\} \quad (1)
\]

\[
\max_{q_j} (a - Q - w)q_j, \forall j \in \{m + 1, m + 2, \ldots, n_d\} \quad (2)
\]

By taking first order condition of the above profit functions and taking into account the symmetric equilibrium of \( q_i \)'s and \( q_j \)'s, the equilibrium conditions for downstream production stage can be derived

\[
(a - t) - (m + 1)q_i - (n_d - m)q_j = 0 \quad (3)
\]

\[
a - mq_i - (n_d - m + 1)q_j = w \quad (4)
\]

Then we can derive the decision of downstream firms and market price as follows

\[
q_i = \frac{(a - t) + (w - t)(n_d - m)}{1 + n_d}
\]

\[
q_j = \frac{(a + mt) - w(m + 1)}{1 + n_d}
\]

\[
Q = \frac{n_d(a - w) + m(w - t)}{1 + n_d}
\]

\[
p_d = \frac{(a + mt) + (n_d - m)w}{1 + n_d}
\]

Then we turn back to the upstream stage. The market demand for the upstream product comes from the \( n_d - m \) non integrated downstream firms. Certainly, those \( n_u - m \) vertically separated upstream firms will supply intermediary products. The other \( m \) vertically integrated firms may potentially supply the intermediary product to the markets and have net sales of \( s_i, \forall i \in [1, m] \).

The total quantity supplied in upstream market is

\[
Q_u = \sum_{j=m+1}^{n_u} q_{uj} + \sum_{i=1}^{m} s_i
\]

Since by the one-to-one production technology adopted in final good production, the
derived demand to upstream product is

\[ Q_u = (n_d - m)q_j \]

\[ = (n_d - m)[(a + mt) - w(1 + m)] \]

\[ (1 + n_d) \]

The the inverse demand to upstream products is

\[ w = \frac{1}{1 + m}[(a + mt) - \frac{1 + n_d}{n_d - m} \left( \sum_{j=m+1}^{n_u} q_{u_j} + \sum_{i=1}^{m} s_i \right)] \] (5)

Then we can solve upstream equilibrium by the following equilibrium condition.

\[ \Delta_{11}s + \Delta_{12}q_u = \Delta_{13} \] (6)

\[ \Delta_{21}s + \Delta_{22}q_u = \Delta_{23} \] (7)

where

\[ \Delta_{11} = n_dm^2 + n_d + 3m^2 + 2m + 1 \]

\[ \Delta_{12} = -n_dm - n_d - n_d - an_d + n_d - 3n_d - n_d - m + n_u - 3m^2 \]

\[ \Delta_{13} = -(mt + an_d - an_d - tn_d + mt - am - m^2t + am^2) \]

\[ \Delta_{21} = (1 + n_d)m \]

\[ \Delta_{22} = (1 + n_d)(n_u - m + 1) \]

\[ \Delta_{23} = (a - t)(n_d - m) \]

The above equalities are derived respectively from the standard first-order condition of integrated and non-integrated upstream firms. The solution to these two equalities are discussed in the Appendix A.

We denote \((s_i(m, n_d, n_u), q_{u_j}(m, n_d, n_u))\) as the unique solution to (6) and (7).

The equilibrium upstream price can be represented as

\[ w(m, n_d, n_u) = \frac{1}{1 + m}[(a + mt) - \frac{1 + n_d}{n_d - m} \left( (n_u - m)q_{u_j}(m, n_d, n_u) + ms_i(m, n_d, n_u) \right)] \] (8)

Therefore, we can figure out the equilibrium profit of an integrated firm

\[ \hat{\pi}(m, n_d, n_u) = [a - mq_i - (n_d - m)q_j - t]q_i + (w(m, n_d, n_u) - t)s_i(m, n_d, n_u) \] (9)

Similarly, the joint profit of an upstream and downstream firms is

\[ \pi_d + \pi_u = [a - mq_i - (n_d - m)q_j - w(m, n_d, n_u)]q_i + (w(m, n_d, n_u) - t)q_{u_j}(m, n_d, n_u) \] (10)

where

\[ \begin{align*}
q_i &= \frac{(a-t)+w(m,n_d,n_u)-t)(n_d-m)}{1+n_d}, \quad \forall i \in \{1,2,\ldots,m\} \\
q_j &= \frac{(a+mt)-w(m,n_d,n_u)(m+1)}{1+n_d}, \quad \forall j \in \{m+1,m+2,\ldots,n_u\}
\end{align*} \]

Then we can discuss the integration decision of firms in the integration game.
4 The equilibrium in the vertical integration stage

In this stage, we assume that the firms make their integration decisions by comparing the profit from vertical integration \( \hat{\pi}(m, n_d, n_u) \) and the joint profit when they both stay nonintegrated \( \pi^{vs}(m, n_d, n_u) + \pi^{vs}(m, n_d, n_u) \). These profits under different vertically related scenarios are just calculated and tractable. The idea is supported because once \( \hat{\pi}(m, n_d, n_u) > \pi^{vs}(m, n_d, n_u) + \pi^{vs}(m, n_d, n_u) \), the upstream and downstream firm can have more profits than stay nonintegrated no matter how they split the benefit from vertical integration. Therefore, we can further simplify the actions taken by firms that downstream firms initiate the integration game if \( n_u > n_d \) whereas upstream firm play the integration game if \( n_d > n_u \). To be more specific, downstream firm decides to integrate one upstream firm if \( \hat{\pi}(m, n_d, n_u) > \pi^{vs}(m, n_d, n_u) + \pi^{vs}(m, n_d, n_u) \) and \( n_u > n_d \) while in the other case upstream firm take the integration decision.

The integration game in this stage will be split into two cases by its possible initial asymmetric market structure. Thus we will consider the first case of equal firms in both upstream and downstream markets (i.e., \( n_d = n_u = n \)) and different number of firms in both markets (i.e., \( n_d \neq n_u \)). The first case gives us much convenience to derive all tractable results for the important reference case. The second case shows us how the relative size in both markets matters for the firms’ integration decision.

4.1 The Symmetric Market setup \( n_d = n_u = n \)

In this case, \( n_d = n_u = n \), we can easily figure out the profit of vertical integration firm in fully vertical integration market structure.

\[
\hat{\pi}(n, n, n) = \frac{(a - t)^2}{(1 + n)^2} 
\]  

(11)

The joint profit of non integration firms is

\[
\pi^{vs}(0, n, n) + \pi^{vs}(0, n, n) = \frac{n(2n + 1)(a - t)^2}{(1 + n)^4} 
\]  

(12)

Then direct subtraction of (11) from (12) lead to

\[
\hat{\pi}(n, n, n) - (\pi^{vs}(0, n, n) + \pi^{vs}(0, n, n)) < 0 \text{ since } \frac{n(2n + 1)}{(1 + n)^2} > 1, \forall n \geq 2
\]

Then the following lemma is obvious.

**Lemma 1.** In the successive n-firm oligopolies, firms are better off in the full vertical separation case than in the full vertical integration case.
In the full vertical integration market, the game is a degenerated n-firm Cournot oligopolistic competition. Each firm in the fully integrated market will have the incentive to over produce comparing with the full vertical separation market. It is because integrated firms in full vertical integration case have less marginal cost in procuring intermediary products than in full vertical separation. The fierce competition in the downstream market drives down each firm’s profit level. Therefore vertical integration will not increase profits, no matter the firm number is.

4.1.1 Market foreclosure and strategic buying

Before we proceed into the further investigation of Nash-equilibrium in the first stage integration game, we discuss the possible market foreclosure effect in the traditional literature (Hart and Tirole, 1990). Here we will use the production stage equilibria derived from asymmetric market setting. In that case, only some but not all firms are integrated (\( m \in \{1, 2, \ldots, \min\{n_d, n_u\} - 1\} \)). We impose the symmetric case here (\( n_d = n_u = n \)) and the market foreclosure effect can be investigated from the solution for \( s_i(m, n, n) \).

\[
s_i(m, n, n) > 0 \iff \frac{m}{n - m} > \frac{3m}{2m^2 - mn - 1} < 0
\]

The detailed solution procedure of \( s_i(m, n, n) \) is shown in the Appendix. Here somewhat surprisingly, the sign of \( s_i(m, n, n) \) is contingent on the ratio of vertically integrated firms to the non integrated firms. \( s_i \) could be positive or negative in the production equilibrium. It is possible that each integrated firm will purchase intermediary input from the non integrated upstream firms at a positive equilibrium market price \( w \) though the integrated firm can procure the same input from its upstream department at a lower marginal cost \( t \). The net selling or net buying of input here serves as strategic complements under the downstream Cournot Oligopoly. A vertically integrated firm \( i \)'s higher net input buying signals higher precommitted production in downstream market. In this situation, the other non integrated downstream firms will reduce their final good production as being a Stackelberg follower. Then the integrated firm \( i \) benefits from the increased demand for its retail final product in the downstream market in the cost of the margin loss in the upstream input market. Another effect is that by doing strategic buying in input market, it drives up the price of the upstream intermediary product for those non integrated downstream firms, therefore ease the competition at the downstream stage. This kind of strategic buying is a case of "bidding up competitors’ costs" (Salop and Sheffman, 1983) and "credible pre-commitment" in the downstream oligopoly (Arya, Mittendorf, and Sappington 2008).
The case that firm makes strategic buying, as is shown in (??), requires that the number of integrated firms be relatively small to the number of non integrated firms. If conversely, the number of non intergraded firms is relatively small, then the effect of strategic buying in easing the downstream competition is limited. Then the integrated firm would not bother to do the strategic buying since the margin loss in input sales is not justified.

On the other hand, the vertically integrated firms will choose to sell inputs to those downstream non integrated firms. The reason is obvious that now non integrated downstream firms do not pose as a sufficiently important part of the downstream market. The relatively small gain from easing the downstream competition is not enough to cover the margin loss in input sales. In a word, the reduction in competition does not generate enough benefits to each integrated firm to make up its expense in doing the "strategic buying." By the derived equilibrium $s_i(m, n_d, n_u)$ in the upstream oligopoly, we can state the following important lemma to conclude this section.

**Lemma 2.** The decision of choosing between the "strategic buying" or "positive input sales" made by each vertically integrated firm reveals

(i) $s_i$ only relates to "the number of vertically integrated firms relative to the non integrated firms" in downstream market. The strategy $s_i(m, n_d, n_u)$ is not affected by market size $a$ and the fixed marginal cost $t$.

(ii) the first vertically integrated firm always market foreclose its downstream rivals, which is $s(m = 1, n_d, n_u) < 0$.

**Proof.** (1) The tedious algebra involved in the derivation of $s_i(m, n_d, n_u)$ is relegated to the Appendix A. Here we know that

$$ (s_i(m, n_d, n_u), q_{u_i}(m, n_d, n_u)) = \left( \frac{(n_d - m)(a - t)\Psi}{(1 + n_d)\Phi} , \frac{(1 + m)(2m + 1 + n_d)(n_d - m)(a - t)}{(1 + n_d)\Phi} \right) $$

where

$$ \Psi = 2m^2 + 3m + mn_d - 2n_u m - n_d - 2n_u - 1 $$

$$ \Phi = n_d - mn_d + n_d n_u + n_d n_u m + 2m^2 + 1 + m + n_u m + n_u $$

By the above equality and the fact that $a - t > 0$, the sign of $S_i$ only relies on the sign of $\Psi$. The sign $\Psi$ is only associated with "the number of vertically integrated firms relative to the non integrated firms" in downstream market.

(2)

$$ s(m = 1, n_d, n_u) = \frac{2(n_d - 1)(1 - n_u)(a - t)}{(n_d + 1)(n_d n_u + n_u + 2)} < 0 $$

since $n_u \geq 2$ by assumption. □
4.1.2 Equilibrium Vertically-related Industrial Structure

In the above discussion, vertically integrated firms may still sell inputs to non integrated firms in the downstream market. Thus the coexistence of vertically integrated and non integrated firms in both downstream and upstream markets may emerge as a possible equilibrium market structure. Here vertically integrated firms have different incentives to market foreclose non integrated firms in the downstream market. The incentives to market foreclose competitors depends on the different initial configuration of the market structure. In the following propositions, we will discuss incentive of vertical integration in different initial industry configuration.

**Proposition 1.** In the symmetric industry setup $n_d = n_u = n$,

(i) full vertical integration $m = n$ is an equilibrium industry structure.

(ii) full vertical separation $m = 0$ is an equilibrium industry structure if and only if $n \geq 5$.

**Proof.** (1) To prove the first part, we begin by proposing a Nash Equilibrium outcome that all firms are vertically integrated and then check if the full vertical integration industry structure is Nash Equilibrium or not.

The full vertical integration profit of an integrated firm is

$$\hat{\pi}(n, n, n) = \frac{(a - t)^2}{(1 + n)^2}, \forall n \geq 2$$

The consolidated profit of non integrated firms in upstream and downstream markets is

$$\pi_v^u(n - 1, n, n) + \pi_v^d(n - 1, n, n) = \frac{(a - t)^2(n^5 - 3n^4 + 13n^3 - 6n^2 - 3n + 4)}{(1 + n)(n^3 + 2n^2 - n + 2)^2}, \forall n \geq 2$$

The direct profit comparison under two different scenarios yields

$$\hat{\pi}(n, n, n) - (\pi_v^u(n - 1, n, n) + \pi_v^d(n - 1, n, n)) = \frac{(a - t)^2nK(n)}{(1 + n)^2(n^3 + 2n^2 - n + 2)^2} \geq 0, \forall n \geq 2$$

since

$$K(n) = (6n^4 - 8n^3 - 7n^2 + 18n - 5)$$

$$K'(n) = 24n^3 - 24n^2 - 14n + 18 > 0, \forall n \geq 2$$

$$K(2) = 35 > 0$$

The vertical integration strategy always increases the joint profits of a pair of firms given all the other pair of firms are already integrated.
(2) To prove the second part, we check if full vertical separation is a Nash equilibrium or not.

\[ \pi_d^{vs}(0, n, n) + \pi_u^{vs}(0, n, n) = \frac{n(2n + 1)(a - t)^2}{(1 + n)^4} \]

\[ \hat{\pi}(1, n, n) = \frac{(a - t)^2(5n^3 + 11n^2 + 27n) - 11}{(1 + n)(2n^2 + 2n + 4)} \]

Direct comparison of the profit is

\[ \pi_d^{vs}(0, n, n) + \pi_u^{vs}(0, n, n) - \hat{\pi}(1, n, n) = \frac{(a - t)^2 I(n)}{4(1 + n)^4(n^2 + n + 2)^2} > 0, \forall n \geq 5 \]

since

\[ I(n) = 3n^6 - 6n^5 - 27n^4 - 56n^3 - 11n^2 + 22n + 11 \]

\[ I'(n) = 18n^5 - 30n^4 - 108n^3 - 168n^2 - 22n + 22 > 0, \forall n \geq 5 \]

\[ I(5) = 4096 > 0 \]

\[ \Box \]

**Proposition 2.** In the symmetric industry setup \( n_d = n_u = n \), full vertical integration \( m = n \) is the only equilibrium industry structure if and only if \( n \leq \bar{n} = 4 \).

**Proof.** Here we just sketch the proof and leave all the tedious algebra and numerical analysis in the Appendix

(i) Following from proposition 2 and lemma 1, vertical integration is not a dominant strategy when \( n \geq 5 \)

(ii) We can check numerically that

\[ \hat{\pi}(m+1, n, n) > \pi_d^{vs}(m, n, n) + \pi_u^{vs}(m, n, n), \forall m \in \{0, 1, \ldots, n-1\} \quad \text{for each of} \quad n \in \{2, 3, 4\}. \]

(iii) Vertical integration is not a dominant strategy \( \forall n \geq 5 \) following directly from the proposition 1 that full vertical separation \( m = 0 \) is a Nash equilibrium.

It follows directly from the definition of Nash Equilibrium that the vertical integration is a dominant strategy when \( n \leq \bar{n} = 4 \). Since vertical integration is the dominant strategy for any possible vertically-related pair, full vertical integration \( m = n \) is the only equilibrium market structure. \[ \Box \]

The threshold of vertical integration is related with the idea of \( s_i(m, n, n) \) adopted here. The non integrated firms now face very strong ”strategic buying” that \( s_i < 0 \). It is because that when \( n < \bar{n} \), \( m \) is big relative to \( n - m \) in the downstream
market. However, as the number of firms increases, the downstream mark-up decreases and the gain from vertical integration falls.

More interesting facts can be inferred from the equilibrium full integration structure. When \( n < \bar{n} \), the market just degenerates to a one stage Cournot Oligopoly, the equilibrium production level turn out to be much larger than the colluded optimal production level. This can somewhat explain that D-Ram industry suffers from greater market price volatility when demand condition is also volatile.

Up to now, these two propositions point out that full vertical integration is the only equilibrium vertically-related industry structure when there are few firms in this vertically-related industry. Therefore the game finally ends up with full vertical integration that all firms choose to be vertically integrated in equilibrium which is the same case as the Semiconductor and LCD industry in Korea.

Semiconductor and LCD industries in Taiwan; however, are featured by many small and medium sized enterprises (SMEs) in the initial market stage of the market. Thus the equilibrium market structure is indeterminate and can not be ensured here since there remains the possibility of many equilibria with \( m \in \{1, 2, \ldots, n - 1\} \). We have already shown the indeterminacy of equilibrium market structures in the above propositions when \( n \geq 5 \).

4.2 The Asymmetric Industry Setup \( n_d \neq n_u \)

We now generalize our model by allowing for asymmetric configuration of firms in downstream and upstream industries. The following two propositions discuss the threshold existing in different market structures: that in which \( n_u > n_d \) and that in which \( n_d > n_u \). We will discuss only the ideas here and left the solution for production and sales decisions of intermediary inputs in the Appendix.

The case where \( n_d > n_u \) will be considered first and then the case where \( n_u > n_d \). The game structure is the same as we specified previously. The two upstream firms simultaneously decide whether to vertically integrate based on the SPE profits of the successive two production stages.

The following lemma shows the first result between an proposed strategy profile and the resulting industry structure.

**Lemma 3.** Given that \( m \) vertically integrated firms in the market, there exists a \((m + 1)\)th pair of vertically-related firms deciding to vertically integrate in this industry if \( \max[n_d, n_u] \geq 5 \).

Larger number of firms in the industry will drive down the individual firm’s profits due to the Cournot competition in this vertically-related industry. Besides, the joint
profits of a pair of vertically separated firms will be lowered as more vertically-integrated firms in the industry since those integrated firms makes the downstream market more competitive and reduce the input demand produced by the independent upstream firms.

This resulting lemma is crucial for us to check the existence and uniqueness of full vertical integration industrial structure in the equilibrium. To check the uniqueness of equilibrium full vertical integration in the initial industry configuration of \( \max[n_d, n_u] \geq 5 \), we only have to check if the first pair of vertically-related firms would emerge in this industry or not. The existence of equilibrium full vertical integration industrial structure can be verified if \( m \geq 1 \) integrated firms in the industry or not. The following propositions show the equilibrium vertically-related industrial structure in the asymmetric industrial configuration.

**Proposition 3.** In the asymmetric industry setup \( n_d > n_u \).

(i) full vertical separation \( m = 0 \) is an equilibrium industrial structure if and only if \( n_d \geq 5 \).

(ii) for every \( n_d \geq 2 \) there exists a \( n_u^* < n_d \) such that full vertical integration \( m = n_u \) is an equilibrium industrial structure if and only if \( n_d > n_u \geq n_u^* \).

**Proposition 4.** In the asymmetric industry setup \( n_d > n_u \), full vertical integration is the only equilibrium industrial structure if and only if \( (n_d = 3, n_u = 2) \) or \( (n_d = 4, n_u = 3) \)

**Proposition 5.** In the asymmetric industry setup \( n_u > n_d \).

(i) full vertical integration \( m = n_d \) is an equilibrium industrial structure.

(ii) for every \( n_d \geq 2 \) there exists a \( n_u^* > n_d \) such that full vertical separation \( m = 0 \) is an equilibrium industrial structure if and only if \( n_d < n_u < n_u^* \).

**Proposition 6.** In the asymmetric industrial setup \( n_u > n_d \), given a \( n_d \geq 2 \) there exists a \( n_u^* \) such that full vertical integration is the only equilibrium industrial structure if and only if \( n_u \geq n_u^* > n_d \)

The reason why firms do vertical integration in this asymmetric market setting is similar to what we derive in the previous symmetric case. However, the situation is much more favorable to vertical integration in the case of \( n_u > n_d \) than in the case of \( n_d > n_u \). In the case of \( n_u > n_d \), the market structure will change dramatically once full vertical integration occurs since there will be no demand for independent upstream firms and they will have no options but to go out off business. The integrated firms will end up control the whole markets. In viewing from the above two propositions, the threshold of vertical integration is much higher in the case of \( n_u > n_d \) than in the case of \( n_d > n_u \). Full vertical integration will be the only equilibrium industrial structure if relative larger firms running business in the upstream side relative to the downstream side. The opportunity cost of vertical integration for upstream firms will be lower as
more firms in the upstream since more upstream firms will drive down the individual firm’s profit in the upstream. By Lemma 2, more upstream firms can effectively reduce the opportunity cost of vertical integration for the first pair of vertically integrated firm in the industry since the first integrated firm by all means market forecloses downstream rivals and incurs a loss in the upstream industry. Thus, more firms in the upstream industry will promotes the emerging of first vertically-integrated firm in the vertically-related industry and promotes the full vertical integration industrial structure by Lemma 3.

4.3 Partial Vertical Integration and Equilibrium Vertically-related Industrial Structures.

An interesting equilibrium industrial structure featured here is the coexistence of $m$ pair of vertically integrated firms and $n_d - m$ independent firms in the downstream industry and $n_u - m$ independent firms in the upstream industry.

This partial vertical integration industrial structure will be a Nash equilibrium if and only if

$$\hat{\pi}(m, n_d, n_u) \geq \pi^{vs}_d(m - 1, n_d, n_u) + \pi^{vs}_u(m - 1, n_d, n_u)$$

and

$$\hat{\pi}(m + 1, n_d, n_u) \leq \pi^{vs}_d(m, n_d, n_u) + \pi^{vs}_u(m, n_d, n_u)$$

The first condition verifies that those firms decide to vertically integrate will not deviate from their integrated status by disintegrating itself into independent downstream and upstream firms. The second condition verifies the nondeviation status of those pairs of the vertically separated firms.

The following lemma shows the result between an proposed initial market setup and the resulting industry structure.

**Proposition 7.** $n_d = 4, n_u = 2$ and $m = 1$ is the only equilibrium partial vertical integrated industrial structure in the vertical integration game.

The equilibrium partial vertical integrated industrial structure does not exist in either $n_d \geq 5$ or $n_u \geq 5$ of initial industry configuration by Lemma 3. Therefore, numerical analysis within $n_d \in \{2, 3, 4, 5\}$ and $n_u \in \{2, 3, 4, 5\}$ will verify the unique partial vertical industrial structure in the equilibrium. Then we can draw the Figure 3 by all the equilibrium industrial structures revealed by proposition 1 to proposition 7.

Figure 3 is drawn to illustrate the equilibrium vertical industrial structures in different initial industry configuration. Full vertical integration can emerge as an equilibrium if both upstream and downstream firms are few and equal numbered or the number of upstream firms are relatively much larger than the number of downstream firms. The multiple equilibria in the vertical integration game reflects the relatively few and equal
Figure 2: The Equilibrium Industry Structure of vertical integration game. Cases of $n_u \leq 10$ & $n_d \leq 10$. 
number of industrial setup initially. The unique partial vertical integration equilibrium can only be generated in the case of larger downstream firms where relatively few vertically independent firms in upstream industries can stay more profitable in comparison with the vertically-integrated status. If the downstream firms want to integrated the upstream counterpart, then it should bear the huge opportunity cost from upstream side since the first vertically integrated firm entering the markets by strategic buying in the upstream markets and generates a net loss in the upstream department. Therefore, a relatively larger number of downstream firms to upstream firms can guarantee the nonexistence of vertically integrated firm in the equilibrium industrial structure. The unique partial vertical integration case lies in the border of multiple equilibria and the full vertical separation area. The relatively larger number of \( n_d = 4 \) to \( n_u = 2 \) entails a mild loss of upstream department when the only vertically integrated firm entering the market by market foreclosing the other three rivals in the downstream industry. However the equilibrium industry structure jumps to full vertical separation when the number of downstream firms get higher.

5 Conclusion

The contrasting difference in the industrial structure in Korea and Taiwan is specifically addressed in this paper. We explain the different market formation of IT manufacturing industries in South Korea and Taiwan. The equilibrium industrial structure is endogenized to be the results of various initial industry configurations. The polar cases of full vertical integration (separation) is characterized by the relatively few downstream (upstream)firms to upstream (downstream) firms. Multiple equilibria exists in the transition of full vertical integration to full vertical separation. That is, both full vertical integration and separation can arise in an equilibrium when the numbers of upstream and downstream firms are close. Therefore, the results revealed here that South Korea IT manufacturing industries should have relatively few firms in both upstream and downstream. Conversely, Taiwan IT market is featured by the initial fragmented market structure in both upstream and downstream. Besides, the coexistence of vertical integration and outsourcing business fits the empirical facts in IT manufacturing industries. Compared with the empirical facts, South Korea government only supports those giant Chaebol producers since the late 80s. The South Korea government adopt both of the following strategies to support the growth of domestic IT manufacturing industries. The close to zero interest and other extremely favorable industrial policies to only a few firms jointly decrease the number of incumbent firms as well as the cost to make vertical integration decision. Our model explains one facet of market formation in this two contrasting markets. This model may shed some light on future research of industrial
policies.

References


Appendix

A. The derivation of $s_i(m, n_d, n_u)$ and $q_{uj}(m, n_d, n_u)$

First, we know that demand for upstream product is given by

$$Q_u = (n_d - m)q_j = \frac{(n_d - m)((a + mt) - w(1 + m))}{1 + n_d}$$

Rearrange the above demand condition, yields the inverse demand of intermediary good

$$w = \frac{1}{1 + m}[(a + mt) - \frac{(1 + n_d)}{n_d - m}Q_u] = \frac{1}{1 + m}[(a + mt) - \frac{(1 + n_d)}{n_d - m}\left(\sum_{j=m+1}^{n_u} q_{uj} + \sum_{i=1}^{m} s_i\right)]$$

The second equality follows from the fact that intermediary products are supplied by the vertically integrated firms $i \in \{1, 2, \ldots, m\}$ and vertically separated firms, $j \in \{m + 1, m + 2, \ldots, n_u\}$.

Then the integrated and separated firms in upstream market maximize their respective profits by

$$\pi_{ui} = \max_{s_i} (a - Q - t)q_i + (w - t)s_i, \forall i \in \{1, 2, \ldots, m\} \quad (A1)$$

$$\pi_{uj} = \max_{q_{uj}} (w - t)q_{uj}, \quad \forall j \in \{m + 1, m + 2, \ldots, n_d\} \quad (A2)$$

with the following demand conditions derived in the downstream market.

$$\begin{align*}
q_i &= \frac{(a - t) + (w - t)(n_d - m)}{1 + n_d}, & \forall i \in \{1, 2, \ldots, m\} \\
q_j &= \frac{(a + mt) - w(m + 1)}{1 + n_d}, & \forall j \in \{m + 1, m + 2, \ldots, n_d\} \\
Q &= mq_i + (n_d - m)q_j = \frac{n_d(a - w) + m(w - t)}{1 + n_d},
\end{align*}$$

and the demand for upstream product,

$$w = \frac{1}{1 + m}[(a + mt) - \frac{(1 + n_d)}{n_d - m}\left(\sum_{j=m+1}^{n_u} q_{uj} + \sum_{i=1}^{m} s_i\right)]$$

Thus, the first order conditions to (A) and (A) are

$$\begin{align*}
\Delta_{11}s + \Delta_{12}q_{u} &= \Delta_{13} \quad (A3) \\
\Delta_{21}s + \Delta_{22}q_{u} &= \Delta_{23} \quad (A4)
\end{align*}$$
where

\[ \Delta_{11} = n_d m^2 + n_d + 3m^2 + 2m + 1 \]
\[ \Delta_{12} = -n_d n_u - n_d m^2 + n_d m - a n_d + n_d n_u m + 3n_a m - m + n_u - 3m^2 \]
\[ \Delta_{13} = -(mtn_d + an_d - an_d m - tn_d + mt - am - m^2 t + am^2) \]
\[ \Delta_{21} = (1 + n_d) m \]
\[ \Delta_{22} = (1 + n_d) (n_u - m + 1) \]
\[ \Delta_{23} = (a - t) (n_d - m) \]

We can use the Cramer’s Rule to solve the unique solution

\[ (s_i(m, n_d, n_u), q_{u_j}(m, n_d, n_u)) = \left( \frac{(n_d - m)(a - t) \Psi}{(1 + n_d) \Phi}, \frac{(1 + m)(2m + 1 + n_d)(n_d - m)(a - t)}{(1 + n_d) \Phi} \right) \]

where

\[ \Psi = 2m^2 + 3m + mn_d - 2n_u m - n_d - 2n_u - 1 \]
\[ \Phi = n_d - mn_d + n_d n_u + n_d n_u m + 2m^2 + 1 + m + n_a m + n_u \]

We can verify that the denominator is positive:

\[ \Phi = n_d - mn_d + n_d n_u + n_d n_u m + 2m^2 + 1 + m + n_a m + n_u > 0 \quad \text{since} \quad m < n_u \]

Obviously, the numerator in \( q_{u_j} \) is positive, which assures that \( q_{u_j} \) is positive.

As for the sign of \( s_i(m, n_d, n_u) \), it can be determined by the sign of \( \Psi \). Take the critical point of \( \Psi = 0 \) and plugging in \( n_u = n_d = n \), yields

\[ 2m^2 + 3m - mn - 3n - 1 = 0 \]
\[ 3(m - n) = mn + 1 - 2m^2 \]
\[ \frac{m}{n - m} = \frac{3m}{2m^2 - mn - 1} \]

Then the sign of \( s_i(m, n_d, n_u) \) is ensured by

\[ s_i(m, n_d, n_u) > 0 \iff \frac{m}{n - m} > \frac{3m}{2m^2 - mn - 1} \]
\[ < 0 \iff \frac{m}{n - m} < \frac{3m}{2m^2 - mn - 1} \]

when \( n = n_d = n_u \) is imposed.
B. Proof of Proposition 2

(i) Following from the proposition 1, vertical integration is not a dominant strategy if and only if
\[ n \geq 5 \]

(ii) We can check numerically that
\[ \hat{\pi}(m + 1, n, n) > \pi_d^{us}(m, n, n) \quad \forall m \in [0, n - 1] \quad \text{and} \quad n = 2, 3, 4 \]

By plugging equilibrium \( q_i, q_j, w(m, n_d, n_u), s_i(m, n_d, n_u), q_u \) into(??) and (??), we can get the equilibrium profit
\[ \hat{\pi}(m, n_d, n_u) \quad \text{and} \quad \pi_d^{us}(m, n_d, n_u) + \pi_u^{us}(m, n_d, n_u) \]

Then for all \( n_d \in [2, 4] \) we list the pay off matrix of \( \hat{\pi}(m, n_d, n_u) \) and \( \pi_d^{us}(m, n_d, n_u) + \pi_u^{us}(m, n_d, n_u) \) as follows. The numbers in the following tables are in units of \( (a-t)^2 \).

(1) \( n = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 2, 2) )</td>
<td>0.165</td>
<td>0.11</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 2, 2) + \pi_u^{us}(m, 2, 2) )</td>
<td>0.123</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n = 2 \).

(2) \( n = 3 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 3, 3) )</td>
<td>0.096</td>
<td>0.085</td>
<td>0.062</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 3, 3) + \pi_u^{us}(m, 3, 3) )</td>
<td>0.082</td>
<td>0.066</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n = 3 \).

(3) \( n = 4 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 4, 4) )</td>
<td>0.061</td>
<td>0.058</td>
<td>0.050</td>
<td>0.04</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 4, 4) + \pi_u^{us}(m, 4, 4) )</td>
<td>0.057</td>
<td>0.049</td>
<td>0.035</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n = 4 \).

By the above (1),(2)and (3), vertical integration is a dominant strategy when \( n = n_d = n_u \leq 4 \)
C. Proof of Proposition 4

By plugging equilibrium \( q_i, q_j, w(m, n_d, n_u), s_i(m, n_d, n_u), q_{u_j} \) into (??) and (??) we can get the equilibrium profit

\[ \hat{\pi}(m, n_d, n_u) \text{ and } \pi_d^{us}(m, n_d, n_u) + \pi_u^{us}(m, n_d, n_u) \]

Then for all \( n_d \geq 2 \) we list the pay off matrix of \( \hat{\pi}(m, n_d, n_u) \) and \( \pi_d^{us}(m, n_d, n_u) + \pi_u^{us}(m, n_d, n_u) \) as follows. The numbers in the following tables are in units of \( (a - t)^2 \).

(1) \( n_d = 3, n_u = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 3, 2) )</td>
<td>0.13</td>
<td>0.105</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 3, 2) + \pi_u^{us}(m, 3, 2) )</td>
<td>0.11</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n_d = 3 \) and \( n_u = 2 \).

(2) \( n_d = 4, n_u = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 4, 2) )</td>
<td>0.111</td>
<td>0.101</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 4, 2) + \pi_u^{us}(m, 4, 2) )</td>
<td>0.106</td>
<td>0.109</td>
</tr>
</tbody>
</table>

Since \( 0.101 < 0.109 \), vertical integration is not a dominant strategy when \( n_d = 4 \) and \( n_u = 2 \). Therefore, (1) and (2) shows that \( n_d = 4, n_u = 2, m = 1 \) is an equilibrium partial integration in the vertical integration game.

(3) \( n_d = 4, n_u = 3 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 4, 3) )</td>
<td>0.0756</td>
<td>0.0712</td>
<td>0.0568</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 4, 3) + \pi_u^{us}(m, 4, 3) )</td>
<td>0.0725</td>
<td>0.0647</td>
<td>0.0452</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n_d = 4 \) and \( n_u = 3 \).

Thus proves the proposition 4.

D. Proof of Proposition 5

For all \( n_u \geq 2 \) we list the pay off matrix of \( \hat{\pi}(m, n_d, n_u) \) and \( \pi_d^{us}(m, n_d, n_u) + \pi_u^{us}(m, n_d, n_u) \) as follows. The numbers in the following tables are in units of \( (a - t)^2 \).

(1) \( n_u = 3, n_d = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 2, 3) )</td>
<td>0.139</td>
<td>0.111</td>
</tr>
<tr>
<td>( \pi_d^{us}(m, 2, 3) + \pi_u^{us}(m, 2, 3) )</td>
<td>0.104</td>
<td>0.067</td>
</tr>
</tbody>
</table>

24
Hence vertical integration is a dominant strategy when \( n_d = 2 \) and \( n_u = 3 \).

(2) \( n_u = 4, n_d = 2 \)

\[
\begin{array}{c|cc}
 m = 0 & m = 1 \\
\hline
\hat{\pi}(m + 1, 2, 4) & 0.128 & 0.111 \\
\pi^u_d(m, 2, 4) + \pi^u_u(m, 2, 4) & 0.098 & 0.067 \\
\end{array}
\]

Hence vertical integration is a dominant strategy when \( n_d = 2 \) and \( n_u = 4 \).

(3) \( n_u = 4, n_d = 3 \)

\[
\begin{array}{c|ccc}
 m = 0 & m = 1 & m = 2 \\
\hline
\hat{\pi}(m + 1, 3, 4) & 0.083 & 0.077 & 0.063 \\
\pi^u_d(m, 3, 4) + \pi^u_u(m, 3, 4) & 0.070 & 0.056 & 0.036 \\
\end{array}
\]

Hence vertical integration is a dominant strategy when \( n_d = 3 \) and \( n_u = 4 \).

(4) \( n_u = 5, n_d = 2 \)

\[
\begin{array}{c|cc}
 m = 0 & m = 1 \\
\hline
\hat{\pi}(m + 1, 2, 5) & 0.123 & 0.111 \\
\pi^u_d(m, 2, 5) + \pi^u_u(m, 2, 5) & 0.096 & 0.069 \\
\end{array}
\]

Hence vertical integration is a dominant strategy when \( n_d = 2 \) and \( n_u = 5 \).

(5) \( n_u = 5, n_d = 3 \)

\[
\begin{array}{c|ccc}
 m = 0 & m = 1 & m = 2 \\
\hline
\hat{\pi}(m + 1, 3, 5) & 0.076 & 0.072 & 0.063 \\
\pi^u_d(m, 3, 5) + \pi^u_u(m, 3, 5) & 0.064 & 0.052 & 0.036 \\
\end{array}
\]

Hence vertical integration is a dominant strategy when \( n_d = 3 \) and \( n_u = 5 \).

(6) \( n_u = 5, n_d = 4 \)

\[
\begin{array}{c|cccc}
 m = 0 & m = 1 & m = 2 & m = 3 \\
\hline
\hat{\pi}(m + 1, 4, 5) & 0.054 & 0.052 & 0.047 & 0.04 \\
\pi^u_d(m, 4, 5) + \pi^u_u(m, 4, 5) & 0.050 & 0.042 & 0.032 & 0.022 \\
\end{array}
\]

Hence vertical integration is a dominant strategy when \( n_d = 4 \) and \( n_u = 5 \).

(7) \( n_u = 6, n_d = 2 \)

\[
\begin{array}{c|cc}
 m = 0 & m = 1 \\
\hline
\hat{\pi}(m + 1, 2, 6) & 0.119 & 0.111 \\
\pi^u_d(m, 2, 6) + \pi^u_u(m, 2, 6) & 0.095 & 0.073 \\
\end{array}
\]
Hence vertical integration is a dominant strategy when $n_d = 2$ and $n_u = 6$.

(8) $n_u = 6, n_d = 3$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 3, 6)$</td>
<td>0.072</td>
<td>0.069</td>
<td>0.062</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 3, 6) + \pi_u^{us}(m, 3, 6)$</td>
<td>0.061</td>
<td>0.050</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 3$ and $n_u = 6$.

(9) $n_u = 6, n_d = 4$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 4, 6)$</td>
<td>0.050</td>
<td>0.049</td>
<td>0.045</td>
<td>0.04</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 4, 6) + \pi_u^{us}(m, 4, 6)$</td>
<td>0.046</td>
<td>0.038</td>
<td>0.031</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 4$ and $n_u = 6$.

(10) $n_u = 6, n_d = 5$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 5, 6)$</td>
<td>0.0376</td>
<td>0.0374</td>
<td>0.0350</td>
<td>0.0316</td>
<td>0.0278</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 5, 6) + \pi_u^{us}(m, 5, 6)$</td>
<td>0.0374</td>
<td>0.0321</td>
<td>0.0266</td>
<td>0.0212</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 5$ and $n_u = 6$.

(11) $n_u = 7, n_d = 2$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 2, 7)$</td>
<td>0.1177</td>
<td>0.1111</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 2, 7) + \pi_u^{us}(m, 2, 7)$</td>
<td>0.0955</td>
<td>0.0759</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 2$ and $n_u = 7$.

(12) $n_u = 7, n_d = 3$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 3, 7)$</td>
<td>0.070</td>
<td>0.068</td>
<td>0.062</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 3, 7) + \pi_u^{us}(m, 3, 7)$</td>
<td>0.059</td>
<td>0.050</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 3$ and $n_u = 7$.

(13) $n_u = 7, n_d = 4$

<table>
<thead>
<tr>
<th>$m$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\pi}(m + 1, 4, 7)$</td>
<td>0.047</td>
<td>0.047</td>
<td>0.044</td>
<td>0.04</td>
</tr>
<tr>
<td>$\pi_d^{us}(m, 4, 7) + \pi_u^{us}(m, 4, 7)$</td>
<td>0.043</td>
<td>0.037</td>
<td>0.031</td>
<td>0.024</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when $n_d = 4$ and $n_u = 7$. 

26
(14) \( n_u = 7, n_d = 5 \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 5, 7) )</td>
<td>0.035</td>
<td>0.035</td>
<td>0.033</td>
<td>0.031</td>
<td>0.028</td>
</tr>
<tr>
<td>( \pi_d^{vs}(m, 5, 7) + \pi_u^{vs}(m, 5, 7) )</td>
<td>0.034</td>
<td>0.029</td>
<td>0.025</td>
<td>0.021</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Hence vertical integration is a dominant strategy when \( n_d = 5 \) and \( n_u = 7 \).

(15) \( n_u = 7, n_d = 6 \)

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\pi}(m + 1, 6, 7) )</td>
<td>0.027</td>
<td>0.028</td>
<td>0.026</td>
<td>0.025</td>
<td>0.023</td>
<td>0.020</td>
</tr>
<tr>
<td>( \pi_d^{vs}(m, 6, 7) + \pi_u^{vs}(m, 6, 7) )</td>
<td>( 0.029 )</td>
<td>0.025</td>
<td>0.022</td>
<td>0.018</td>
<td>0.015</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Since \( 0.027 < 0.029 \), vertical integration is not a dominant strategy when \( n_u = 7 \) and \( n_d = 6 \).

Thus proves the proposition 5.

(16) Since we have numerically check the equilibrium industrial structure up to \( n_d = 7 \). Thus, given a \( n_d \geq 7 \) we have to show that there exists a \( n_u^* \) such that full vertical integration is the only equilibrium industrial structure if and only if \( n_u \geq n_u^* > n_d \).

\[
\hat{\pi}(1, n_d, n_u) = \frac{(a - t)^2(4n_u^2 + 4n_dn_u^2 + 16n_u - 11 + 7n_d^2 + 11n_d + n_d^3)}{(2n_dn_u + 4 + 2n_u)^2(1 + n_d)}
\]

\[
\pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u) = \frac{(a - t)^2(n_d^2 + n_u^2 + n_d)}{(1 + n_d)^2(1 + n_u)^2}
\]

Given that \( n_u \geq n_d \), We first choose the min\( \{n_u\} = n_d \) such that

\[
\hat{\pi}(1, n_d, n_u) = \frac{(a - t)^2(11n_u^2 + 5n_u^3 - 11 + 27n_u)}{(2n_u^2 + 4 + 2n_u)^2(1 + n_u)}
\]

\[
\pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u) = \frac{(a - t)^2(2n_u^2 + n_u)}{(1 + n_u)^4}
\]

\[
\Rightarrow \hat{\pi}(1, n_d, n_u) < \pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u) \quad \text{if} \quad n_d \geq 5
\]

\[
\Rightarrow \frac{\partial \hat{\pi}(1, n_d, n_u)}{\partial n_u} < 0
\]

\[
\Rightarrow \frac{\partial (\pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u))}{\partial n_u} < 0 \quad \text{iff} \quad n_u < n_d^2 + n_d
\]

\[
\Rightarrow \frac{\partial (\pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u))}{\partial n_u} > 0 \quad \text{iff} \quad n_u > n_d^2 + n_d
\]
By the illustration of Figure 4, \( \hat{\pi}(1, n_d, n_u) > (\pi_u^{vs}(0, n_d, n_u) + \pi_d^{vs}(0, n_d, n_u)) \) if and only if \( n > n^* \). Then the proof is done by directly applying Lemma 3 that \( \hat{\pi}(m + 1, n_d, n_u) > (\pi_u^{vs}(m, n_d, n_u) + \pi_d^{vs}(m, n_d, n_u)) \) \( \forall m \in \{1, 2, \ldots, n_d\} \)