Contagion of Liquidity Crisis between Two Firms

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This paper presents a model in which the contagion of a liquidity crisis between two non-financial institutions occurs due to the learning within a common creditor pool. After creditors observe what occurs in a firm’s rollover game, they conjecture each other’s "type," or the attitude toward the risk of a firm’s investment project. Creditors’ inference of others’ types then affects their own decisions with regard to the next firm that they lend to. Through the analysis of each firm’s "incidence of failure" – the threshold for a liquidity crisis – this paper demonstrates that the risk of contagion rises in an important way if originating from a firm that ex-ante faces a small probability of failure. This paper also offers policy proposals to mitigate the severity of contagion in such liquidity crises. (JEL G33, G38, D82, D83)

Key Words: Contagion; Liquidity crisis; Global games; Learning; Collateral; Government’s bailout; Information structure; Financial disclosure

1. INTRODUCTION

Contagion is a propagation of the solvency problems of a single institution to other institutions. It is one of the most striking features of a financial crisis, in that it causes the crisis to spread across countries and institutions. In the late 1990’s, most East Asian countries suffered severe financial crises via contagion across countries, the so-called "Asian Flu." When South Korea suffered the Asian Flu, the liquidity crisis spread from one firm to other firms even though their businesses were not closely related. For example, in January 1997, Hanbo Steel Group – the country’s fourteenth-largest conglomerate – went bankrupt, and within a few months, Jinro – the largest liquor group in Korea – failed.1 They had connections with each other only via common bank creditors. How do we explain these kinds of serial (contagious) failures of non-financial firms whose businesses are not related to each other?

In this paper, I present a model in which the contagion of a liquidity crisis between two unrelated non-financial institutions occurs due to the co-creditors’ learning about each other’s "type," or the attitude toward the risk of a firm’s investment project. Some fairly extensive studies deal with a contagion of the financial crisis among financial institutions and/or international financial markets based on their interlinkages and changes in asset prices (Allen and Gale (2000) and Cifuentes, Ferrucci, and Shin (2005) among others).2 However, studies on the contagion of the liquidity crisis among non-financial businesses have received only scant attention.

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1This 1997 Korean financial crisis will be explained more specifically in section 5.
My aim is to contribute to the understanding of the contagion phenomenon among non-financial institutions based on the idea that the contagion’s triggering mechanism is the learning within a common creditor pool. Specifically, I suggest that when co-creditors learn about each other’s "type," contagion is triggered.

I focus on the "self-fulfilling crisis" – a crisis that occurs just because creditors believe it is going to occur. The self-fulfilling nature of the crisis is important because a liquidity crisis in a firm is often viewed as resulting from a coordination failure among creditors. However, to approach the nature of the crisis as self-fulfilling tends to produce multiple equilibrium outcomes, and thus it is hard to demonstrate the contagion effect. Therefore, to obtain the unique equilibrium (the threshold for a liquidity crisis), I employ the global games method introduced by Carlsson and van Damme (1993). This method allows me to get the unique equilibrium in each firm and therefore capture the contagion effect in which a liquidity crisis in one firm affects the likelihood of a crisis in another.

Specifically, the global games setting of the firm and of the creditors is similar to the one used by Morris and Shin [M-S] (2004). M-S (2004) analyze the coordination game in the debt market by using tools of global games. They show that the creditors of a distressed borrower face a coordination problem (a rollover game among creditors). They further demonstrate that, without common knowledge on the fundamentals of the distressed borrower, the incidence of failure is uniquely determined, given that the creditors’ private information on the fundamentals is precise enough. However, they just tackle one firm’s rollover game among one type of creditors and do not cover the contagion of the liquidity crisis among firms, which is the central issue of my paper. I extend M-S (2004)’s model to the case of two firms with two different types of creditors. By doing so, I explain the contagion phenomenon between two firms.

For the contagion setting between two firms, I generally refer to Goldstein and Pauzner [G-P] (2004). G-P (2004) use the global games method to explain the contagion phenomenon between two countries. They look at two countries that have independent fundamentals but share the same group of investors. In their model, a crisis in one country reduces agents’ wealth, which makes them more averse to the strategic risk associated with the unknown behaviors of other agents in the second country. This increases agents’ incentive to withdraw their investments in the second country. That is, the mechanism that generates the contagion in their model originates in a wealth effect. However, in my paper, I focus on the creditors’ learning about each other’s type as the contagion mechanism. In a coordination game setting, the learning process is very important because it can directly explain the creditors’ strategic behaviors, which in turn affect the probability of the liquidity crisis in the firm.

As Goldstein and Pauzner (2004) mention in their paper, models with multiple equilibria cannot capture the contagion effect in which a liquidity crisis in one firm affects the likelihood of a liquidity crisis in the other firm because they do not predict the likelihood of each particular equilibrium.


Kyle and Xiong (2001) also explain the contagion of the financial crisis between two countries based on the wealth effect, even though they do not use the global games approach.

Angeletos, Hellwig, and Pavan (2007) study how learning about the underlying fundamentals influences the dynamics of coordination in a global game of regime change. Similarly, Manz (2002) shows that the failure of a single firm can trigger a chain of failures when investors learn about a common state influencing all firms within an industry, such as a proxy variable for the demand...
I examine a sequential framework in which the rollover game among creditors in firm \( A \) takes place before it occurs in firm \( B \). Creditors in my model hold loans for two firms' investment projects.\(^7\) In each firm, they can either roll over their loans until maturity, in which case they can get full repayment from the firm if the investment project succeeds, or they can recall their loans in the interim stage, in which case they can get some premature liquidation value (collateral debt) but less than the full repayment amount. The success of the investment project depends on the fundamentals of the firm and on the number of creditors in that firm who keep rolling over their loans until maturity. That is, creditors' coordination on whether or not to roll over the loans determines the likelihood of a liquidity crisis in the firm.

There are two types of creditors: one is "pessimistic" and the other is "optimistic."\(^9\) "Pessimistic" creditors worry about the failure of the firm's investment project more than "optimistic" creditors do. In practice, the different types reflect both the strength of the balance sheet (the financial status) of each creditor and any information advantage on firm-related issues, including the economic situation. That is, a creditor with a weak balance sheet and/or with an information disadvantage on firm-related issues holds a more "pessimistic" attitude toward the risk he takes than one who has a strong balance sheet and/or an information advantage on firm-related issues.

Following the global games method, I assume that creditors do not have common knowledge on the fundamentals of firm \( A \) and firm \( B \). Rather, creditors get noisy signals about the firm’s fundamentals after they are realized. In this setting, based on the private signals about the firm’s fundamentals, different types of creditors uniquely determine both their own beliefs on the fundamentals of each firm and their own actions on whether or not to roll over the loans until maturity in that firm. After the rollover game in firm \( A \) ends, creditors observe the aggregate outcomes of firm \( A \), which depend on firm \( A \)'s fundamentals and on creditors’ actions in firm \( A \).

Observing what occurred in firm \( A \), creditors can conjecture other creditors’ types since the outcome of the rollover game in firm \( A \) depends on the different actions of different type creditors. Hence, before the rollover game in firm \( B \) occurs, creditors can revise their beliefs about other creditors’ types. After learning about facing the products of all firms. In my paper, however, I examine how learning about the types of other co-creditors plays a role as the contagion mechanism of the liquidity crisis between two firms. Taketa (2004a) analyzes the contagion phenomenon of currency crises between two countries using the global games method with the learning process of speculators. However, he does not numerically analyze the contagion effect and its severity. Focusing on non-financial institutions, I specifically analyze the contagion effects and suggest policy proposals to reduce the severity of contagion on the liquidity crisis from one firm to the other.

\(^7\) Chen (1999) shows that the systemic risk may occur in the absence of any interbank relations due to the first-come, first-served rule and to information externalities on the negative payoffs. That is, he models banking panics as the outcome of "information-based herding behavior" by depositors. However, the global games approach that I use in this paper has a quite different mechanism from the herding model. Morris and Shin (2003) differentiate the two as follows: *The global games analysis is driven by strategic complementarities and the highly correlated signals generated by the noisy observations technology. However, the sensitivity to the information structure arises in a purely static setting. The herding stories have no payoff complementarities and simple information structures, but rely on sequential choice.*

\(^8\) Co-creditors, for example, can be thought of as common bank creditors of different firms.

\(^9\) Izmalkov and Yildiz (2010) show that in strategic environments the relevant measure of sentiments (i.e., pessimistic / optimistic outlook) can vary arbitrarily and have a large impact on the strategic behavior even when there is little uncertainty.
other creditors’ types from the outcome of firm A, creditors uniquely determine their beliefs on the fundamentals of firm B and their actions in firm B. If there is a liquidity crisis in firm A, and if firm B also suffers the liquidity crisis due to the creditors’ learning process, then there is a "contagion" of the liquidity crisis from firm A to firm B. Moreover, I refer to the increased probability of the liquidity crisis in firm B due to the contagion as a "severity of contagion" on the liquidity crisis.

Having shown the severity of contagion on the liquidity crisis from firm A to firm B, I demonstrate that the severity of contagion is greater when the originating firm’s "failure point" – the probability that the firm’s investment project will fail – has decreased. In other words, the liquidity crisis in a firm that has a small possibility of failing is more contagious than otherwise. This is a striking result compared with other contagion-related papers, which deal with contagion among international financial markets and/or financial institutions through capital linkages and asset price changes. In these papers, the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected through their linkages.

Also, I analyze the policy implications of reducing the severity of a liquidity crisis contagion from firm A to firm B. Firm B can minimize the severity of a contagion from firm A to itself by setting the value of its collateral small, since the decreased value of the collateral is the increased cost of not rolling over the loans from the creditors’ standpoint. The government can also play a role to reduce the severe contagion damage of the liquidity crisis by making the pessimistic creditors more optimistic about the success of the firm’s investment project (e.g., by providing bailouts to the firm that suffers a transitory liquidity problem) and by reducing the degree of incomplete information on the creditors’ types in the market (e.g., by implementing the financial disclosure policy that discloses the types of creditors).

Regarding creditors’ information structure, I find that increasing the accuracy of creditors’ information on the firm’s fundamentals lowers the failure point of the individual firm. However, in the same way that the severity of contagion is more serious when the originating firm’s failure point is lower, the severity of contagion is also more serious when creditors have more accurate information. That is, if the liquidity crisis occurs in the firm considered less likely to fail (i.e., the firm with a small failure point because creditors have precise information structure on the fundamentals), then it leads to a big surprise in the market, and thus the liquidity crisis can be more contagious. Based on this phenomenon, I argue that policies promoting transparency and precise information on the firm’s fundamentals are not a panacea in a crisis episode. Even though transparency reduces the probability of a crisis in one economy’s case, it worsens the severity of contagion among more than one economy.

The remainder of this paper is as follows. I present the model in section 2. In section 3, I solve for firm A’s equilibrium and firm B’s equilibrium, and show how the contagion of the liquidity crisis from firm A to firm B occurs through the creditors’ learning process. In section 4, I define "severity of contagion" on the liquidity crisis and discuss some policy implications to reduce this severity. In section 5, I discuss the applicability of my model to real-world phenomena, focusing on Korea’s 1997 financial crisis. Section 6 concludes.
2. MODEL

There are two firms: firm A and firm B. Both firms own no capital, and their investment projects are only financed by loans from creditors. There are two groups of creditors: group 1 and group 2. The order of events (see figure 1) is as follows.\(^{10}\)

First, nature determines what the creditors are like. Second, creditors lend their money to both firms A and B. Third, the states of each firm’s fundamentals (\(\theta_A\) and \(\theta_B\)) are realized. Fourth, each creditor in each group (\(j = 1, 2\)) receives a private signal (\(x_{Aj}\)) on the fundamentals of firm A. Fifth, each creditor decides whether or not to roll over the loan in firm A. Sixth, the exact realization of the fundamentals of firm A and the result of the creditors’ actions (i.e., firm A’s project failure or success) are known to all creditors after the rollover game in firm A ends.\(^{11}\) Seventh, each creditor in each group (\(j = 1, 2\)) gets a private signal (\(x_{Bj}\)) on the fundamentals of firm B. Eighth, each creditor decides his action in firm B. Ninth, the exact realization of the fundamentals of firm B as well as the aggregate behaviors in firm B are known to all creditors.

Creditors are financing both investment projects of two firms: firm A and firm B. In other words, two firms share the same creditors. There are two groups of creditors: group 1 and group 2, both consisting of a continuum of small creditors, so that any individual creditor’s stake is negligible as a proportion of the whole.\(^{12}\)

\(^{10}\)I generally follow G-P (2004)’s sequence. Note that the model is sequential: the activity takes place in firm A and then in firm B.

\(^{11}\)That is, before creditors decide on their actions, they did not know the exact value of the firm’s fundamentals. However, I assume that after the rollover game ends, creditors get to know the true value of the firm’s fundamentals. As G-P (2004) mention in their paper, in equilibrium, it is sufficient that creditors receive information regarding either the fundamentals or the aggregate behaviors of creditors since one can be inferred from the other.

\(^{12}\)Corsetti, Dasgupta, Morris, and Shin [C-D-M-S] (2004) use the global games approach to consider the implication of the existence of a large speculator like George Soros in a currency crisis in the dynamic setting. But they do not cover the contagion effect there. Based on C-D-M-S (2004), Taketa (2004b) analyzes the implication of the presence of a large speculator in contagious currency crises: making countries more vulnerable to crises but mitigating the contagion of crises across countries. In my paper, for simplicity, I focus purely on all small players in the static / simultaneous game setting.
I assume that all creditors are in a unit interval $[0, 1]$. The size of group 1 is $\lambda$ and that of group 2 is $(1 - \lambda)$, where $0 < \lambda < 1$. There exists uncertainty about the creditors’ type, that is, about the creditors’ attitudes toward the risk (bullishness) of a firm’s investment project. Thus, group 1’s type is its own private information. There are two possible types of group 1 creditors: "pessimistic," with probability $q$, and "optimistic" with probability $(1 - q)$. For simplicity, group 2’s type is "pessimistic" and is public information to all creditors. I assume that the type of each group remains the same, without big exogenous shocks such as government’s intervention or an entire breakdown of the market.

"Pessimistic" creditors worry about the failure of the firm’s investment project more than "optimistic" creditors do. The different types reflect both the strength of the balance sheet (the financial status) of each creditor and any information advantage on firm-related issues, including the economic situation. That is, a creditor with a weak balance sheet and/or with an information disadvantage on firm-related issues holds a more "pessimistic" attitude toward the risk he takes than one who has a strong balance sheet and/or an information advantage on firm-related issues. I assume that "pessimistic" creditors use $\delta_P$ as their discount factor, which is less than $\delta_O$ – the discount factor of "optimistic" creditors (i.e., $0 < \delta_P < \delta_O < 1$). That is, "pessimistic" (bearish) creditors put less present value on the firm’s investment project than "optimistic" (bullish) creditors do.

The state of firm $i$’s fundamentals is $\theta_i$, where $i = A, B$. $\theta_i$ can be interpreted as a measure of the ability of firm $i$ to meet short-term claims from creditors. The higher value of $\theta_i$ refers to the better fundamentals. $\theta_i$ is randomly drawn from the real line after both firms raise funds from creditors and invest the funds in their projects. I assume that $\theta_A$ and $\theta_B$ are independent, which means that there is no linkage of fundamentals between firm $A$ and firm $B$.

After $\theta_i$ ($i = A, B$) is realized, the rollover game among creditors takes place in sequences: firm $A$ first and then firm $B$. In each firm’s rollover game, there are two periods: period 1 (interim stage) and period 2 (maturity), in which creditors lend for a firm’s investment project. The investment project of each firm is completed in period 2 and yields the return $v_i$ ($i = A, B$), which is uncertain initially because it depends on the creditors’ actions in period 1. Financing of firm $A$ and firm $B$ is undertaken by a standard debt contract. For simplicity, I assume that both firms have the same debt contract. That is, the face value of the repayment is $L$, and each creditor receives this full amount in period 2 if the realized value of $v_i$ is large.

13That is, nature randomly chooses the type of group 1 creditors: "pessimistic" or "optimistic." Group 1 creditors know their own type, but group 2 creditors do not know the type of group 1 creditors. Group 2 creditors can just expect that the type of all group 1 creditors is "pessimistic" with probability $q$ or "optimistic" with probability $(1 - q)$. However, group 1’s type can be revealed to group 2 creditors after the rollover game in firm $A$ ends, which I tackle in section 3.2.

14Instead of the "pessimistic" type, I can set the type of group 2 creditors as being "optimistic." It does not affect the contagion result of my model because the type of group 2 creditors is public information in the market. Of course, the type of group 2 creditors affects the probability of the liquidity crisis in each individual firm.

15In practice, a creditor’s financial status can change over time and his informativeness is different for firm $A$ and firm $B$. In my work, for simplicity, I assume that a creditor’s financial status does not change in the course of the model’s timeline and that his informativeness for two firms is the same.

16This two-period rollover game among creditors is directly based on M-S (2004)’s model.

17In general, firms use various debt contracts and they can screen the types of creditors. However, in my model, I explain creditors’ learning process about each other’s type by simply focusing on the standard debt contract. Analyzing creditors’ learning process, I define the contagion of the liquidity crisis from firm $A$ to firm $B$ in section 3.3.
enough to cover the repayment of debt.

At period 1, before the final realization of \( v_i \), the creditors have an opportunity to review their investment. Hence, in this period, creditors have to decide whether or not to roll over their loans until period 2. The loans are collateralized, and if creditors collect and liquidate the collateral after they do not roll over the loans (period 1), the liquidation value of the seized collateral is \( K^* \in (0, L) \). However, if the creditors collect and liquidate the collateral because they cannot get the full repayment after they roll over the loans (period 2), the liquidation value of the seized collateral is \( K_* \), which is less than \( K^* \) (i.e., \( K_* < K^* < L \)). That is, if I denote the proportion of creditors who do not roll over the loans of firm \( i \) at period 1 by \( l_i \) \((i = A, B)\), then the firm’s investment project fails if and only if \( l_i > \theta_i \) and creditors get \( K_* \) at period 2.\(^{18}\)

As M-S (2004) do, for the simplicity of my discussion, I normalize the payoffs so that \( L = 1 \) and \( K_* = 0 \). Then, creditors who do not roll over the loans at period 1 get \( K \), which is in \((0, 1)\).\(^{19}\) In summary, the present values of the payoffs at period 1 to a creditor are given by the following matrix:

<table>
<thead>
<tr>
<th>Rollover</th>
<th>Project succeeds</th>
<th>Project fails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not rollover</td>
<td>( \frac{\delta_m \cdot 1 = \delta_m}{K} )</td>
<td>( \frac{\delta_m \cdot 0 = 0}{K} )</td>
</tr>
</tbody>
</table>

where \( m \) is \( P \) for a "pessimistic" creditor or \( O \) for an "optimistic" creditor. I assume \( 0 < K < \delta P < \delta O < 1 \).

If the creditors know the value of \( \theta_i \) perfectly before deciding whether or not to roll over the loans (period 1), their optimal strategies are like Obstfeld (1996)’s self-fulfilling story, as follows. If \( \theta_i > 1 \), then creditors will roll over their loans irrespective of other creditors’ actions because the project survives even if every other creditor recalls. Conversely, if \( \theta_i \leq 0 \), then it is optimal for creditors not to roll over the loans since the state of the fundamentals of the firm is so bad that the project will fail even if all other creditors roll over their loans. When \( \theta_i \in (0, 1) \), the coordination problem among the creditors occurs. If all other creditors roll over their loans, then the payoff to rolling over the loan is 1 at maturity (period 2)\(^{20}\), so that rolling over the loan yields more than the premature liquidation value \( K \). Meanwhile, if everyone else recalls the loan, then the payoff is 0, which is less than \( K \), so that early liquidation is optimal. Hence, the common knowledge assumption of creditors on \( \theta_i \) leads to multiple equilibria.\(^{21}\)

To get the unique equilibrium, I apply the global games method here: \( \theta_i \) is not the common knowledge. Rather, at period 1 when creditors decide whether or not to roll over the loans, they receive private information concerning \( \theta_i \), but it is not perfect. In other words, each creditor in group \( j \) \((j = 1, 2)\) gets the private signal:

\[^{18}\] The firm remains in operation given that \( \theta_i \) is large enough to meet the claims from creditors. Otherwise, it is pushed into default. Specifically, if \( \theta_i \geq l_i \), then the firm’s investment project succeeds and the realized value of \( v_i \) is equal to \( V \) which is a constant greater than \( L \). Meanwhile, if \( l_i > \theta_i \), then the project fails and \( v_i = K_* \).

\[^{19}\] The exact value of \( K \) is \( \frac{K^* - K_*}{L - K_*} \) by normalizing the payoffs, and it is in \((0, 1)\) since \( K_* < K^* < L \).

\[^{20}\] At period 1, the present value of 1 is \( \delta P \) for "pessimistic" creditors or \( \delta O \) for "optimistic" creditors.

\[^{21}\] As M-S (2004) mention in their paper, this type of coordination problem among creditors is analogous to the bank run problem of Diamond and Dybvig [D-D] (1983). However, D-D (1983) do not cover contagion issues. They just focus on analyzing the coordination failure among patient depositors in one bank and show the result of multiple equilibria.
$x_{ij} = \theta_i + \varepsilon_{ij}$, where \( \varepsilon_{ij} \) is uniformly distributed over the interval \([-\varepsilon, \varepsilon]\). Note that the creditor’s present value (at period 1) of the expected utility of rolling over the loan based on his private signal is $U = \delta_m \cdot \Pr \{ \theta_i \geq l_i \mid x_{ij} \}$, where $m = P$ or $O$, and that of recalling the loan is $K$. A strategy for the creditor is a decision rule which maps each realization of $x_{ij}$ to an action – rolling over the loan or not rolling over the loan. An equilibrium strategy consists of (1) a firm’s switching fundamentals ($\bar{\theta}_i$) below which the project fails (i.e., a liquidity crisis occurs in the firm) and (2) the creditors’ switching private signal ($\bar{x}_{ij}$) such that every creditor who receives a signal lower than $\bar{x}_{ij}$ does not roll over the loan.\(^{22}\)

In the following section, I solve for the equilibrium strategy of firm A ($\bar{\theta}_A$ and $\bar{x}_{Aj}$, where $j = 1, 2$) first. After the rollover game in firm A ends, every creditor observes what occurred in firm A, including the exact value of $\theta_A$. Here, group 2 creditors can conjecture or learn the "type" of group 1 creditors based on the outcome in firm A (i.e., whether a liquidity crisis in firm A occurred or not) and on firm A’s switching fundamentals. Next, I solve for the equilibrium strategy of firm B ($\theta_B$ and $\bar{x}_{Bj}$, where $j = 1, 2$), which is affected by creditors’ revised beliefs – which are formed after the rollover game in firm A ends – about other creditors’ types. This explains how and why a liquidity crisis in firm A can trigger a liquidity crisis in firm B (i.e., it explains a contagion of the liquidity crisis from firm A to firm B).

3. SOLVING THE MODEL

3.1. Equilibrium in Firm A

Firm A’s equilibrium strategy consists of (1) a firm’s switching fundamentals (\(\bar{\theta}_A\)) below which the project fails (i.e., a liquidity crisis occurs in firm A) and (2) the creditors’ switching private signal (\(\bar{x}_{Aj}\)) such that every creditor who receives a signal lower than $\bar{x}_{Aj}$ does not roll over the loan. Here, the equilibrium values $\bar{\theta}_A$ and $\bar{x}_{Aj}$ are as follows:

\[\bar{\theta}_A = \begin{cases} 
\theta_{AP} & \text{if the type of group 1 creditors is "pessimistic";} \\
\theta_{AO} & \text{if the type of group 1 creditors is "optimistic";}
\end{cases}\]

\[\bar{x}_{A1} = \begin{cases} 
x_{A1P} & \text{if group 1 creditors are "pessimistic";} \\
x_{A1O} & \text{if group 1 creditors are "optimistic";}
\end{cases}\]

\[\bar{x}_{A2} = x_{A2}^{*}.
\]

After getting a private signal in period 1, each creditor has to decide whether or not to roll over the loan. The indifference condition gives the following equation:

\[
K \underbrace{\text{payoff from recalling}}_{\text{PV of the payoff from successful rollover}} = \delta_m \cdot \Pr \{ \text{rollover is successful} \mid \bar{x}_{Aj} \}. \tag{1}
\]

Also, note that the critical threshold value of firm A’s fundamentals (i.e., switching fundamentals) is determined when the proportion of creditors who do not roll

\(^{22}\)M-S (2004) consider both private and public signals on the firm’s fundamentals. In my paper, for simplicity, I just assume that creditors get the private signals on the firm’s fundamentals.

\(^{23}\)As M-S (1998, 2003, 2004) discuss in the literatures, even if $\varepsilon$ becomes very small, the realization of $\varepsilon_{ij}$ will not be common knowledge among the creditors. Moreover, in this case, M-S (1998, 2003, 2004) and C-D-M-S (2004) show that the equilibrium strategy consists of a unique value of a firm’s switching fundamentals and a unique value of the creditors’ switching private signal.
over the loans ($l_A$) is equal to $\theta_A$. Using equation (1) for each creditor and the condition of the critical threshold value of firm $A$’s fundamentals, I calculate the unique equilibrium values: the switching fundamentals of firm $A$ ($\theta^*_AP$ and $\theta^*_AO$) and the switching private signals ($x^*_{A1P}$, $x^*_{A1O}$, and $x^*_{A2}$). Firm $A$’s equilibrium is summarized in the following proposition.

**Proposition 1.** There exists a unique equilibrium strategy in firm $A$ that consists of (1) a firm’s switching fundamentals ($\theta_A$) below which the project fails (i.e., a liquidity crisis occurs in firm $A$) and (2) the creditors’ switching private signal ($\bar{x}_{Aj}$, $j = 1, 2$) such that every creditor who receives a signal lower than $\bar{x}_{Aj}$ does not roll over the loan. Specifically, firm $A$’s switching fundamentals are

$$
\theta^*_AP = \frac{K}{\delta_P} (1 - \Sigma_1),
$$

$$
\theta^*_AO = \frac{K}{\delta_P} (1 - \Sigma_1 - \Sigma_2);
$$

and the creditors’ switching private signals are

$$
x^*_{A1P} = \frac{K}{\delta_P} (1 - \Sigma_1 + \Sigma_3),
$$

$$
x^*_{A1O} = \frac{K}{\delta_P} \left( 1 - \Sigma_1 - \Sigma_2 + \frac{\delta_P \Sigma_3}{\delta_O} \right),
$$

$$
x^*_{A2} = \frac{K}{\delta_P} (1 - \Sigma_1 - (1 - q) \Sigma_2 + \Sigma_3),
$$

where

$$
\Sigma_1 = \frac{\lambda (1 - \lambda) (1 - q) (\delta_O - \delta_P)}{\delta_O (1 + 2 \varepsilon - \lambda)}, \quad \Sigma_2 = \frac{2 \lambda \varepsilon (\delta_O - \delta_P)}{\delta_O (1 + 2 \varepsilon - \lambda)} \quad \text{and} \quad \Sigma_3 = \left( \frac{2K - \delta_P}{K} \right) \varepsilon.
$$

**Proof.** See the Appendix.

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**Figure 2:** Firm $A$’s Switching Fundamentals

Liquidity crisis occurs in firm $A$ if the type of group 1 creditors is “pessimistic”

Liquidity crisis occurs in firm $A$ if the type of group 1 creditors is “optimistic”
Creditors in group 1 ("pessimistic type") do not roll over the loans in firm A
Creditors in group 2 do not roll over the loans in firm A
Creditors in group 1 ("optimistic type") do not roll over the loans in firm A

Figure 3: Creditors' Switching Private Signals in Firm A

Note that \( \theta^*_AP > \theta^*_AO \) and \( x^*A1P > x^*_A2 > x^*_A1O \) hold since \( \lambda, q, \) and \( \varepsilon \) are in \((0,1)\), and \( 0 < \delta_P < \delta_O < 1 \) (see figure 2 and figure 3). The intuition of the inequalities is the following. \( x^*_A1P \) is greater than \( x^*_A1O \) because the pessimistic creditors are more likely not to roll over the loans than optimistic creditors. By the same logic, \( \theta^*_AP \) is greater than \( \theta^*_AO \) because firm A’s project will be more likely to fail (i.e., will be liquidated early) if group 1 creditors are pessimistic.

3.2. Equilibrium in Firm B

Now every creditor observes what occurred in firm A, including the exact value of \( \theta_A \). This conveys information about the type of group 1 creditors to the market because different types use different switching signals, resulting in different outcomes in firm A under certain conditions.

There are two possible scenarios. First, if \( \theta_A \notin [\theta^*_AO, \theta^*_AP] \), then the type of group 1 creditors is not revealed. Why? If \( \theta_A \leq \theta^*_AO \), then the liquidity crisis certainly occurs in firm A regardless of the type of group 1 creditors. Meanwhile, if \( \theta_A \geq \theta^*_AP \), then the liquidity crisis never occurs in firm A regardless of the type of group 1 creditors. Hence, if \( \theta_A \notin [\theta^*_AO, \theta^*_AP] \), group 2 creditors do not get to know the type of group 1 creditors and face the same rollover game, which was played in firm A, in determining whether or not to roll over the loans in firm B.

Next, however, if \( \theta_A \in [\theta^*_AO, \theta^*_AP] \), then the type of group 1 creditors is revealed to the market. Conditional on such \( \theta_A \), the liquidity crisis occurs in firm A if and only if group 1 creditors are pessimistic. Likewise, conditional on such \( \theta_A \), which is between \( \theta^*_AO \) and \( \theta^*_AP \), the liquidity crisis does not occur in firm A if and only if group 1 creditors are optimistic. Hence, if \( \theta_A \in [\theta^*_AO, \theta^*_AP] \), then the new rollover

\textsuperscript{24} Note that in this case (\( \theta_A \notin [\theta^*_AO, \theta^*_AP] \)), even though the number of creditors who did not roll over their loans is known, the type of group 1 creditors is not revealed since \( x^*_A1 \) is in the \( \varepsilon \)-neighborhood of \( \theta_A \); and \( x^*_A1P \) and \( x^*_A1O \) are very closely located around \( \theta^*_AP \) and \( \theta^*_AO \), respectively.
game is played by creditors determining whether or not to roll over the loans in firm B.

In the following, I explain the two scenarios: $\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$ and $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$. In each scenario, I derive the equilibrium strategy (i.e., $\theta_B$ and $x_{Bj}$, $j = 1, 2$).

### 3.2.1. Scenario 1: $\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$ 

In this scenario, the type of group 1 creditors is not revealed. Hence, the equilibrium values of the switching fundamentals of firm B and the switching private signals are exactly the same as those of firm A. This is the benchmark case of firm B, and particularly, the benchmark switching fundamentals of firm B are (1) $\theta_{AO}^*$ if group 1 creditors are optimistic, and (2) $\theta_{AP}^*$ if group 1 creditors are pessimistic.

### 3.2.2. Scenario 2 - 1: Liquidity crisis in firm A when $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ 

This scenario implies that the type of group 1 is "pessimistic." In this case, creditors in both group 1 and group 2 have the same switching strategy signal (say $x_B^*$). Hence, the equilibrium strategy consists of (1) a firm’s switching fundamentals ($\theta_{BP}^*$) below which the project fails (i.e., a liquidity crisis occurs in firm B) and (2) the creditors’ switching private signal ($x_{Bj}^*$) such that every creditor who receives a signal lower than $x_B^*$ does not roll over the loan. Here, I get the following equilibrium strategy:

$\theta_{BP}^* = \frac{K}{\delta_P}$,

$x_B^* = \frac{K}{\delta_P} (2 \varepsilon + 1) - \varepsilon$.

### 3.2.3. Scenario 2 - 2: No liquidity crisis in firm A when $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ 

This scenario implies that the type of group 1 is "optimistic." In this case, creditors in both group 1 and group 2 have different switching strategy signals (say $x_{B1}^*$ for group 1 and $x_{B2}^*$ for group 2). Hence, the equilibrium strategy consists of (1) a firm’s switching fundamentals ($\theta_{BO}^*$) below which the project fails (i.e., firm B suffers a liquidity crisis) and (2) the creditors’ switching private signals ($x_{B1}^*$ for group 1 and $x_{B2}^*$ for group 2) such that every creditor in group 1 who receives a signal lower than $x_{B1}^*$ does not roll over the loan and that every creditor in group 2 who receives a signal lower than $x_{B2}^*$ does not roll over the loan. Here, I get the following equilibrium strategy:

$\theta_{BO}^* = \frac{\lambda K}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P}$,

$x_{B1}^* = \frac{K (\lambda + 2 \varepsilon)}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P} - \varepsilon$,

$x_{B2}^* = \frac{\lambda K}{\delta_O} + \frac{K (1 - \lambda + 2 \varepsilon)}{\delta_P} - \varepsilon$.

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25 The derivation is in the Appendix.

26 The derivation is in the Appendix.
Note that $\theta_{AO} > \theta_{BP}$ and $x_{B1} < x_{B2} < x^*_B$ hold since $\lambda$ and $\varepsilon$ are in $(0, 1)$, and $0 < \delta_P < \delta_O < 1$. The intuition of the inequalities is the following. $x^*_B$ is greater than $x_{B1}$ and $x_{B2}$ because when all creditors are pessimistic, they are more likely not to roll over the loans than when there exist optimistic creditors. By the same logic, $\theta_{BP}^*$ is greater than $\theta_{BO}$ because firm $B$'s project will be more likely to fail (i.e., will be liquidated early) if group 1 creditors are pessimistic. Now, firm $B$'s equilibrium is summarized in the following proposition.\footnote{The proof of this result directly follows from the derivation of firm $B$'s equilibrium strategy in each scenario and the same logic as the proof of Proposition 1 given in the Appendix.}

**Proposition 2.** Conditional on the realized underlying state of the fundamentals of firm $A$ ($\theta_A$) and whether a liquidity crisis occurs in firm $A$ or not, there exists a unique equilibrium in firm $B$.

1. When the realized underlying state of the fundamentals of firm $A$ ($\theta_A$) is not in the interval $[\theta_{AO}, \theta_{AP}]$, the same equilibrium values (switching fundamentals and switching private signals) as those of firm $A$ are obtained irrespective of whether a liquidity crisis occurs in firm $A$ or not;

2. When $\theta_A \in [\theta_{AO}, \theta_{AP}]$ and there is a liquidity crisis in firm $A$, every creditor in any group does not roll over the loan if his signal $x_{Bj}$ ($j = 1, 2$) is below $x^*_B$ and does roll over the loan if the signal is above;

3. When $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ but there is no liquidity crisis in firm $A$, each creditor in group 1 does not roll over the loan if his signal $x_{B1}$ is below $x_{B1}^*$ and each creditor in group 2 does not roll over the loan if his signal $x_{B2}$ is below $x_{B2}^*$.

### 3.3. Contagion of the Liquidity Crisis from Firm $A$ to Firm $B$

#### 3.3.1. What is contagion?

In this paper, contagion is defined as a propagation of the solvency problems between two firms, and the contagion of the liquidity crisis from firm $A$ to firm $B$ is propagated by creditors who determine whether or not to roll over the loans. After observing what happened in firm $A$, creditors update their beliefs about other creditors’ types and reflect this information in their optimal decisions in firm $B$.

If the realized value of the fundamentals of firm $A$ ($\theta_A$) is quite bad, which means $\theta_A \leq \theta_{AO}$, then firm $A$ suffers a liquidity crisis regardless of the type of group 1 creditors. In this case, the type of group 1 creditors is not revealed. So if $\theta_A \leq \theta_{AO}$, it does not cause the contagion of the liquidity crisis from firm $A$ to firm $B$ because group 2 creditors’ decisions in firm $B$ are not affected by the situation in firm $A$. Only when $\theta_A$ is between $\theta_{AO}^*$ and $\theta_{AP}^*$ and when there is a liquidity crisis in firm $A$, can I discuss whether there is a contagion of the liquidity crisis from firm $A$ to firm $B$.

As I discussed in section 3.2, if $\theta_A \in [\theta_{AO}, \theta_{AP}]$ and there is no liquidity crisis in firm $A$, this implies that the type of group 1 creditors is "optimistic." This information is reflected in group 2 creditors’ decisions, and $\theta_{BO}^*$ is determined. Likewise, if $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ and there is a liquidity crisis in firm $A$, this implies that the type of group 1 creditors is "pessimistic." This information is reflected in group 2 creditors’ decisions, and $\theta_{BP}^*$ is determined. That is, only when $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$, does the behavior of creditors in firm $A$ affect the behavior of creditors in firm $B$. 
Now if the realized value of the fundamentals of firm B ($\theta_B$) is quite bad, which means $\theta_B \leq \theta_{BO}^*$, then firm B suffers a liquidity crisis regardless of the occurrence of the liquidity crisis in firm A. Hence in this case, even though there are liquidity crises in both firms, I cannot say that there is an actual contagion of the solvency problems from firm A to firm B. Meanwhile, if $\theta_B$ is between $\theta_{BO}^*$ and $\theta_{BP}^*$ and there is a liquidity crisis in firm B, then this is the contagion of the liquidity crisis from firm A to firm B since there can be the liquidity crisis in firm B in $\theta_B \in [\theta_{BO}^*, \theta_{BP}^*]$ only when there was the liquidity crisis in firm A in $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$ (see figure 4).

**Definition 1.** Contagion of the liquidity crisis from firm A to firm B is that there is a liquidity crisis in firm B due to creditors’ learning when $\theta_B \in [\theta_{BO}^*, \theta_{BP}^*]$; and there is a liquidity crisis in firm A when $\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$.

![Contagion of Liquidity Crisis from Firm A to Firm B](image)

**Figure 4: Contagion of Liquidity Crisis from Firm A to Firm B**

### 3.3.2. Scenario 1 versus scenario 2

Now, let’s compare scenario 1 ($\theta_A \notin [\theta_{AO}^*, \theta_{AP}^*]$) with scenario 2 ($\theta_A \in [\theta_{AO}^*, \theta_{AP}^*]$). Scenario 1 provides the benchmark switching fundamentals ($\theta_{AO}^*$ and $\theta_{AP}^*$) in firm B. Meanwhile, scenario 2 provides the new switching fundamentals ($\theta_{BO}^*$ and $\theta_{BP}^*$) in firm B. By comparing the values of these switching fundamentals, I get the following lemma (see figure 5).

**Lemma 1.** $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$.

**Proof.** From the values of $\theta_{AO}^*$, $\theta_{AP}^*$, $\theta_{BO}^*$, and $\theta_{BP}^*$, I get

$$\theta_{BP}^* - \theta_{AP}^* = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left( \frac{1 - \lambda}{1 + 2\varepsilon - \lambda} \right) > 0,$$

$$\theta_{AO}^* - \theta_{BO}^* = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left( \frac{q (1 - \lambda)}{1 + 2\varepsilon - \lambda} \right) > 0.$$

From the fact that $\theta_{AP}^* - \theta_{AO}^* > 0$, $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$ hold.  

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The intuition of the inequalities is as follows. If the type of group 1 creditors is revealed and is "optimistic," then the liquidity crisis is less likely to occur in firm $B$ compared to the case where the type is not revealed (i.e., $\theta_{BO}^* < \theta_{AO}^*$). Meanwhile, if the type of group 1 creditors is revealed and is "pessimistic," then the liquidity crisis more likely occurs in firm $B$ compared to the case where the type is not revealed (i.e., $\theta_{BP}^* > \theta_{AP}^*$). That is, if the type of group 1 creditors is revealed, the possibility of whether the liquidity crisis occurs or not in firm $B$ will become more clear than if the type is not revealed. From this result, I argue that the revelation of the type of group 1 creditors is not always good for firm $B$. It depends on the realized type (i.e., "pessimistic" or "optimistic") of group 1 creditors.

4. COMPARATIVE STATICS AND POLICY IMPLICATIONS

In this section, after defining the severity of contagion on the liquidity crisis, I show that the impact of the contagion originating from the firm considered less likely to fail is bigger than otherwise. Then, by doing comparative statics on the severity of contagion, I suggest some policy implications to reduce the severity of contagion.

4.1. Severity of Contagion on the Liquidity Crisis

In section 3.3.2, I showed that $\theta_{BO}^* < \theta_{AO}^* < \theta_{AP}^* < \theta_{BP}^*$. What does this imply? This means that if the type of group 1 creditors is revealed as being "pessimistic," then the probability of a liquidity crisis in firm $B$ is increased by the difference between $\theta_{BP}^*$ and $\theta_{AP}^*$. This is a negative effect of the contagion on the liquidity crisis in firm $B$. If the type of group 1 creditors is revealed as being "optimistic," then the probability of a liquidity crisis in firm $B$ is decreased by the difference between $\theta_{AO}^*$ and $\theta_{BO}^*$. This can be interpreted as a positive effect of reducing the probability of a liquidity crisis in firm $B$ via the revelation of the optimistic type of group 1 creditors. Focusing on the negative effect of the contagion on the liquidity crisis in firm $B$, I define the severity of contagion as the difference between the new switching fundamentals for pessimistic-type creditors ($\theta_{BP}^*$) and the benchmark switching fundamentals for pessimistic-type creditors ($\theta_{AP}^*$).

**Definition 2.** Severity of contagion on the liquidity crisis in firm $B$ is the increased probability of a liquidity crisis in firm $B$ due to the negative effect of the contagion: the difference between the new switching fundamentals $\theta_{BP}^*$ and the benchmark switching fundamentals $\theta_{AP}^*$. Specifically, it is expressed by

\[
SC := \theta_{BP}^* - \theta_{AP}^* = \frac{\lambda(1 - \lambda)(\delta_O - \delta_P)(1 - q)K}{\delta_O\delta_P(1 + 2\varepsilon - \lambda)},
\]

28That is, I define the severity of contagion on the liquidity crisis in firm $B$ as an increase in the probability of a liquidity crisis in firm $B$ due to the creditors' learning from the liquidity crisis in firm $A$. 

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Figure 5: Values of Switching Fundamentals (Firm $B$)
which is greater than 0 since \( \lambda, \varepsilon, q, K, \delta_O, \) and \( \delta_P \) are in \((0,1)\).  

Now, I get the following proposition.

**Proposition 3.** The liquidity crisis in a firm with a small possibility of failing is more contagious than otherwise.

Proof. I need to show that the severity of contagion \((\theta_{BP}^* - \theta_{AP}^*)\) decreases with \(\theta_{AP}^*\). This is trivial since the decrease of \(\theta_{AP}^*\) will increase the difference between \(\theta_{BP}^*\) and \(\theta_{AP}^*\). Specifically I can express \(\theta_{AP}^*\) as

\[
\theta_{AP}^* = \frac{K}{\delta_P} \left( 1 + \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q)}{\delta_O (-1 - 2\varepsilon + \lambda)} \right)
\]

By arranging the above equation, I get

\[
SC = -\theta_{AP}^* + \frac{K}{\delta_P},
\]

which implies that the severity of contagion \((SC)\) decreases with \(\theta_{AP}^*\).  

This proposition illustrates that the severity of contagion decreases with the level of firm A’s failure point (i.e., firm A’s switching fundamentals). It implies that the occurrence of the liquidity crisis in the firm considered less likely to fail (i.e., the firm having a lower failure point) would lead to a huge surprise in the market, and hence the liquidity crisis is likely to become more contagious than otherwise. In summary, the probable contagion of the liquidity crisis in the firm considered less likely to fail is much bigger than in the firm considered to be not strong enough to endure the liquidity crisis.

This is a noticeable result since other contagion-related papers that deal with contagion among international financial markets and/or financial institutions through capital linkages and asset price changes insist that the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected through their linkages. In my work, however, I find that the severity of contagion is more serious when the originating firm’s failure point is lower. This finding is based on the following conditions: 1) I focus on the co-creditors’ learning process between two non-financial institutions whose businesses

[29] How can I express the positive effect of reducing the probability of a liquidity crisis in firm B due to the revelation of the optimistic type of group 1 creditors? It is \(\theta_{AO}^* - \theta_{BO}^* = \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)}\), which is only different in the term of \(q\) from \((1 - q)\) of \(SC\). That is, the sign of the comparative statics of \(\theta_{AO}^* - \theta_{BO}^*\) with respect to the variables that comprise it is exactly the same as \(SC\)’s case except for \(q\). This brings the trade-off relation of the policy proposals for reducing the severity of contagion, which I tackle in later sections (e.g., the initial policies regarding \(K, \varepsilon, \) and \(\lambda\)). In other words, if the government and/or the firm takes measures to reduce the severity of contagion initially, the positive effect of reducing the probability of a liquidity crisis in firm B due to the revelation of the optimistic type of group 1 creditors is also reduced by those measures. This implies that the effectiveness of the pre-determined policies by the government and/or the firm depends on the type of group 1 creditors.

[30] By the definition of firms’ switching fundamentals, the low value of the switching fundamentals means that the firm fails with a small probability. That is, the value of firm’s switching fundamentals can be interpreted as its failure point.
are not related to each other (i.e., independent fundamentals) as the contagion triggering mechanism; 2) I assume that the exact realization of the fundamentals of the originating firm and the result of creditors’ actions (i.e., the failure or success of the firm’s project) are known to creditors before they determine their actions in the other firm.

4.2. Changes in the Value of the Collateralized Debt (K)

As M-S (2003, 2004) mention in their papers, increasing the value of the collateral (K) has two contrasting effects: first, it increases the value of the debt (loan) in the event of default (i.e., the direct effect\(^{31}\)); second, it increases the range of \(\theta\) at which default occurs (i.e., the strategic effect\(^{32}\)). In the contagion context, I find that the strategic effect outweighs the direct effect, which means that decreasing the value of the collateral (K) is helpful to reduce the severity of contagion on the side of firm B.

**Proposition 4.** Severity of contagion on the liquidity crisis in firm B is reduced by a decrease in the value of its collateral (K).

**Proof.**

\[
\frac{\partial SC}{\partial K} = \frac{\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q)}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)} > 0,
\]

which implies that if firm B decreases the value of K, then the severity of contagion on the liquidity crisis in firm B (SC) will be reduced.

What is the intuition of this proposition? The decreased value of the collateral is the increased cost of not rolling over the loans from the creditors’ standpoint. In other words, creditors have more incentive than otherwise to roll over their loans until maturity when the value of the collateral is small. Hence, firm B can reduce the severity of contagion on the liquidity crisis from firm A to itself by setting the value of its collateral small.

4.3. Changes in the Gap of Discount Factors (\(\delta_O\) and \(\delta_P\))

Diamond and Dybvig (1983) argue in their paper that deposit insurance by the government can prevent bank runs even though it might generate a moral hazard problem. That is, patient agents know that withdrawal by others is not going to harm their long-term return, and they will not withdraw their deposits. Likewise, let’s think about the government bailouts to a firm that suffers a transitory liquidity problem.\(^{33}\) After observing a liquidity crisis in firm A and getting to know that the type of group 1 creditors is pessimistic, the government can expect the contagion of the liquidity crisis from firm A to firm B. If the government provides bailouts to firm B, which is thought of as suffering a transitory liquidity problem even though the state of its fundamentals is not too bad, then it is a good signal for the success of firm B’s investment project in the market. In this case, pessimistic

\(^{31}\)In a similar context, Besanko and Thakor (1987) and Greenbaum and Thakor (2007) argue the signaling issue of the collateral: low-risk borrowers choose contracts with high collateral requirements because their low-risk means that the chance of defaulting and losing the collateral to the creditors is lower; hence, offering the high collateral is less onerous.

\(^{32}\)In my model, I can verify this strategic effect result from \(\theta^*_AP\) and \(\theta^*_BP\).

\(^{33}\)Note that the government’s provision of bailouts is not the full insurance on the success of firm’s investment project.
creditors become more optimistic toward the success of firm B’s investment project (i.e., \( \delta_P \rightarrow \delta_O \)).\(^{34}\) That is, the gap between \( \delta_O \) and \( \delta_P \) decreases, and hence it reduces the severity of contagion on the liquidity crisis in firm B.\(^{35}\) I summarize this argument in the following proposition.

**Proposition 5.** Severity of contagion on the liquidity crisis in firm B is reduced by a decrease in \((\delta_O - \delta_P)\), which is obtained by the government’s provision of bailouts to firm B.

Proof. \[
\frac{\partial SC}{\partial (\delta_O - \delta_P)} = \frac{\lambda (1 - \lambda) (1 - q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)} > 0,
\]
which implies that if the government decreases \((\delta_O - \delta_P)\) by providing bailouts to firm B, then the severity of contagion on the liquidity crisis in firm B (SC) will be reduced. \(\blacksquare\)

However, the government’s bailout policy may have two main problems, as follows. First, the government cannot easily distinguish between the insolvency risk and the illiquidity risk of the firm (Morris and Shin (2009)). As Thakor (2008) argues, the government’s bailout should be primarily intended to stave off the bankruptcy / illiquidity problem and to recover the investors’ sapped confidence. It cannot fix a broken business model. Second, as Fischer (1999) points out, the existence of this "lender-of-last-resort" creates a moral hazard problem with respect to the actions of both creditors and firms. Hence, the government’s bailout policy should be implemented under systematic guidelines, with proper surveillance and regulations, to achieve an improvement in the whole economy’s welfare (Fischer (1999), Gai, Hayes, and Shin (2001), and Schneider and Tornell (2004) among others).

### 4.4. Changes in the Information Structure (\(\varepsilon\))

As creditors’ information on the firm’s fundamentals becomes very precise (i.e., \( \varepsilon \rightarrow 0 \)), the value of the firm’s switching fundamentals is decreased.\(^{36}\) Heinemann and Illing (2002) similarly emphasize the role of transparent / precise information in a crisis episode. Now, what is the effect of small noise (i.e., precise information on the firm’s fundamentals) on the severity of contagion? Will precise information on the firm’s fundamentals reduce the severity of contagion? The result looks very surprising, for I find that it increases the severity of contagion.

**Proposition 6.** Severity of contagion on the liquidity crisis in firm B increases with the accuracy of the information structure.

Proof. \[
\frac{\partial SC}{\partial \varepsilon} = -\frac{2\lambda (1 - \lambda) (\delta_O - \delta_P) (1 - q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)^2} < 0,
\]

\(^{34}\)Here, I just focus on creditors’ optimism toward a firm’s fundamentals. However, in the whole economy’s point of view, if the government implements fiscal and/or monetary expansion policies, then pessimistic creditors become more optimistic and thus \( \delta_P \rightarrow \delta_O \).

\(^{35}\)In the extreme case where the government "fully" guarantees firm B’s investment project, there occurs no contagion of the liquidity crisis from firm A to firm B.

\(^{36}\)In my model, I can verify this result from \( \theta_{AP} \) for instance. By the way, note that \( \theta_{BP} \) and \( \theta_{BO} \) do not have \( \varepsilon \) since the type of group 1 creditors is the known fact when those switching fundamentals are determined.
which implies that if creditors’ (private) information on the firm’s fundamentals becomes very precise (i.e., $\varepsilon \rightarrow 0$), then the severity of contagion on the liquidity crisis in firm $B$ (SC) will be increased.

If creditors’ information on the firm’s fundamentals is very accurate (i.e., if $\varepsilon$ is very small), then the probability of firm $A$’s liquidity crisis is reduced (i.e., the failure point of firm $A$ ($\theta_{AP}$) becomes lower). However, if there occurs the liquidity crisis in firm $A$ even though the probability of the failure is low, then the contagion of the liquidity crisis to firm $B$ is more severe. This can be interpreted as what Proposition 3 addressed. That is, if the liquidity crisis occurs in the firm considered less likely to fail (i.e., the firm having a small failure point ($\theta_{AP}$) via small $\varepsilon$), then it leads to a big shock in the market and thus the liquidity crisis can be more contagious. Based on this result, I argue that the policy for agents’ transparent / precise information on the fundamentals is not a panacea in a crisis episode. Even though transparency reduces the probability of a crisis in the case of one economy, it worsens the severity of contagion in the crisis among more than one economy.

4.5. Changes in the Size of Group 1 ($\lambda$)

The size of group 1 creditors, which is measured by $\lambda$, represents incomplete information in the market. That is, even though the type of group 2 creditors is "pessimistic," which is public information in the market, the type of group 1 creditors is not known in the market initially. What is the effect of this incomplete information on the severity of contagion? In other words, what is the impact of the degree of incomplete information on the contagion? I show the effect of $\lambda$ on the severity of contagion when $\varepsilon$ converges to zero in the following proposition.\(^{37}\)

**Proposition 7.** Severity of contagion on the liquidity crisis in firm $B$ is reduced by a decrease in the size of group $1$.

*Proof.*

\[
\frac{\partial \text{SC}}{\partial \lambda} = \frac{(\delta_O - \delta_P)(1-q)K}{\delta_O \delta_P} > 0 \text{ as } \varepsilon \rightarrow 0,
\]

which implies that as the size of group 1 becomes smaller and as creditors’ (private) information on the firm’s fundamentals becomes very precise, the severity of contagion on the liquidity crisis in firm $B$ (SC) will be decreased. \(\blacksquare\)

What does this proposition imply? As I discussed above, the size of group 1 stands for incomplete information in the market initially. If the size of this incomplete information becomes small, then the contagion of the liquidity crisis becomes less severe. Hence, the government can mitigate the severity of contagion by regulating the size of this incomplete information. For example, the government reinforces creditors to reveal their types via its financial disclosure policy (i.e., to disclose their financial information in the market).\(^{38}\) In the extreme case where the financial disclosure perfectly reveals the type of group 1 creditors in the market, there occurs no learning process among creditors, and thus there is no contagion of a liquidity crisis from firm $A$ to firm $B$.

\(^{37}\)When $\varepsilon$ does not converge to zero, the effect of $\lambda$ on the severity of contagion depends on the relative sizes of $\lambda$ and $\varepsilon$. Hence, here I tackle the case where $\varepsilon$ converges to zero, which means that the information of creditors on the firm’s fundamentals is very precise.

\(^{38}\)Note that even though this revelation policy is helpful to reduce the severity of contagion, it is not always good for the individual firm, which I discussed in section 3.3.2.
Related to the issue of the revelation of the type of group 1 creditors via the financial disclosure policy, what is the effect of the type of group 1 creditors on the severity of contagion? Since the type of group 1 creditors is "pessimistic" with probability \( q \), I find that the severity of contagion decreases with \( q \).\(^{39}\) It implies that if group 2 creditors initially expect that group 1 creditors are more likely the same type as theirs, then the learning process of creditors’ type does not have as much impact on the contagion of the liquidity crisis as otherwise.\(^{40}\)

5. DISCUSSION IN REAL-WORLD PHENOMENA

5.1. Korea’s 1997 Financial Crisis

In order to assess the applicability of my model to real-world phenomena, let us revisit 1997 Korean financial crisis in the middle of the Asian Flu. According to Akama, Noro, and Tada [A-N-T] (2003), Korean firms were highly leveraged by short-term loans from domestic and foreign banks. By the end of 1996, the corporate debt relative to the nominal GDP ratio was over 1.6, and the external debt to the GDP ratio reached approximately 25%, in which the share of short-term debt out of the total external debt peaked at 58%. This fact is parallel to the debt-financing assumption of my model. A-N-T (2003) also argue that Korea had a bank-centered financial system. As of the end of 1997, among 26 domestic commercial banks, 16 nationwide commercial banks\(^{41}\) were actually common bank creditors of the top 30 conglomerates in Korea.\(^{42}\) This means that Korean firms have the same co-lending banks, which is parallel to the co-creditors’ assumption of my model. In summary, the overall business situation of Korean firms at 1997 shows debt-rollover coordinations among co-creditors.

According to Rhee (1998), the bankruptcy of the Hanbo Steel Group in January 1997 was a sobering experience for co-lending banks. They started to strictly reexamine the profitability of their loans on other companies and to call in most of short-term loans. This led to a "domino effect" as more and more companies suffered liquidity crises. For example, Kia Motors – Korea’s eighth-largest conglomerate – failed even though its reputation in the market was fairly good.\(^{43}\) The rush continued, and as I mentioned in the Introduction, Jinro – Korea’s nineteenth-largest conglomerate and also the largest liquor group – failed in September 1997. By the end of 1997, over 15,000 companies, large and small, went bankrupt.\(^{44}\) In the process of serial firms’ failures, we can observe the following phenomenon. Foreign banks (especially, Japanese and U.S. banks\(^{45}\)) pulled out their money en masse,...

\(^{39}\)I check that if \( q < \frac{1}{2} \), \( SC (:=\theta_{BP} - \theta_{AP}) \) is greater than the positive effect of reducing the probability of the liquidity crisis in firm \( B \) due to the revelation of the optimistic type of group 1 creditors: \( \theta_{AO} - \theta_{BO} \).

\(^{40}\)The others were local commercial banks.

\(^{41}\)That is, those commercial banks lent their money to multiple firms, including top 30 conglomerates.

\(^{42}\)In 1998, Kia was merged by Hyundai Motor.

\(^{43}\)As I mentioned above, these companies generally have common banker creditors.

and some Korean domestic banks (e.g., Korea First Bank (KFB)) dramatically stopped rolling over their loans first. Then, other co-lending banks followed to stop rolling over their loans in other firms.

The interpretation of Korea’s 1997 financial crisis is consistent with my model. Observing Hanbo Steel Group’s liquidity crisis, common bank creditors could conjecture or learn other creditors’ types. Here, foreign banks and KFB, for example, can be thought of as pessimistic creditors in my model due to an information disadvantage and a weak balance sheet, respectively. More specifically, foreign banks can be considered as group 2 creditors in my model because they had an information disadvantage on the overall business situations of Korea compared to Korean domestic banks, and this fact was known in the market. Of course, some Korean domestic commercial banks can be treated as group 2 creditors if their bad financial states were known among creditors. In the case of KFB, its financial status was unknown initially, and thus I can interpret KFB as being in group 1 in my model. Reflecting the new information of other banks’ types revealed after Hanbo Steel Group’s liquidity crisis, co-bank creditors decided their own actions—rolling over their loans or not—in other firms.

Noting that there were no fundamental linkages among many firms that went bankrupt; and noting the leading roles of foreign banks and KFB on serial rushes in the market, I clearly argue that Korea’s 1997 financial crisis provides empirical evidence that supports my model of the contagion triggering mechanism: co-creditors’ learning about each other’s type. That is, by learning about other creditors’ types from former firms’ debt-rollover coordinations, creditors determine their own actions toward next firms. Note that Korea’s 1997 financial crisis is different from simple herding stories, which rely solely on the sequential choices of players. It demonstrates the newly repeated static debt-rollover coordination games in other firms among co-creditors via their learning about other creditors’ types from the former firms’ debt-rollover games. Moreover, in the static coordination game setting of each firm, there exist payoff (strategic) complementarities among co-creditors, unlike in the simple herding model.

5.2. Experimental Analysis of the Model

Data constraints on the full financial information of creditors and firms may make it quite difficult to empirically estimate the model. Experimental analysis would then be a good potential work to test my theoretical predictions. In fact, Heinemann, Nagel, and Ockenfels [H-N-O] (2004) design an experiment to test the speculative-attack model of M-S (1998). They conclude that the switching strategy in the theory of global games is an important reference point, providing correct predictions for comparative statics with respect to parameters of the payoff function. Taketa, Suzuki-Löffelholz, and Arikawa [T-S-A] (2009) conduct an experiment designed to imitate the C-D-M-S (2004) model and to support the argument that the presence of a large speculator causes other speculators to be more aggressive in their attacks.

\footnote{KFB went bankrupt right after Jinro’s failure.}

\footnote{In my model of two firms, for simplicity of analysis, I assume that creditors’ types remain the same in the course of two debt-rollover games among creditors. Of course, in the general sequential case of more than two firms, like Korea’s 1997 financial crisis, I should consider the dynamic effects of changes in creditors’ wealth from former firms’ rollover games on changes in creditors’ types. In other words, I need to extend my analysis to include the more dynamic learning process of co-creditors on other creditors’ types with their wealth changes.}
To properly test my model of contagion in an experiment, I need to investigate creditors’ learning behavior about the type of other creditors from the former firm’s debt-rollover coordination game among them on top of the experimental analysis settings of H-N-O (2004) and T-S-A (2009). Hence, I can utilize the "belief-learning" model experiment in which players do not learn about which strategies work best; they learn about what others are likely to do, then use those updated beliefs to change their attractions and hence to change what strategies they choose (Camerer, Ho, and Chong (2001) and Fudenberg and Levine (1998) among others). By doing so in the experiment, I will be able to observe how switching strategies change according to creditors’ learning processes and to test the predictions for comparative statics in my model.

6. CONCLUDING REMARKS

This paper, focusing on liquidity crises in non-financial institutions, explores contagion: the phenomenon that occurs when the states of two firms’ fundamentals are not closely related, but still, what happens in one firm affects the optimal behaviors of creditors and thus what happens in the other firm. The contagion mechanism between two non-financial firms is based on the co-creditors’ learning about each other’s type, which has received little attention from the existing literature. Examining the creditors’ learning process is very important, because in a rollover coordination game, creditors’ beliefs about others’ types affect the probability of the occurrence of the liquidity crisis in the firm, i.e., the creditors’ learning process can be very useful in explaining the creditors’ strategic behaviors in a coordination game. Learning and revising beliefs about others’ types after observing what occurred in one firm, creditors determine their actions in a latter firm, which affects the probability of a liquidity crisis in that latter firm. I discussed the real-world example (i.e., Korea’s financial crisis in 1997), which supports my model.

By analyzing the contagion process with creditors’ learning, I found a noticeable feature of the contagion that is different from previous contagion-related literatures: the contagion impact of the liquidity crisis originating from the firm having a lower failure point is more severe than otherwise under the assumptions that the exact realization of the fundamentals of the firm and the result of creditors’ actions in that firm are known to creditors before they decide their actions in the other firm. Moreover, even though increasing the accuracy of creditors’ information on the firm’s fundamentals reduces the probability of the liquidity crisis in an individual firm, it increases the severity of contagion. I dealt with policy proposals addressing how to mitigate the severity of contagion, including the government’s provision of bailouts to the firm suffering a transitory liquidity problem and its financial disclosure policy. Also, the firm can initially reduce the severity of contagion by setting the value of its collateral small.

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APPENDIX

Proof of Proposition 1

First, let’s think about the decisions of group 1 creditors. They privately know their own type ("pessimistic" or "optimistic") and also know group 2 creditors’ type ("pessimistic"). Hence, they know the value of \( \bar{\theta}_A : \theta_{AP}^* \) or \( \bar{\theta}_A : \theta_{AO}^* \). Note that \( \varepsilon_{A} := x_{A} - \bar{\theta}_A \) is uniformly distributed over the interval \([-\varepsilon, \varepsilon]\). So, the equation (1) becomes:

\[
K = \Pr \left[ \text{rollover is successful} \mid x_{A1}, \bar{\theta}_A \right] \cdot \delta_m
= \Pr \left[ \theta_A \geq \bar{\theta}_A \mid x_{A1}, \bar{\theta}_A \right] \cdot \delta_m
= \Pr \left[ \bar{x}_{A1} - \varepsilon_{A} \geq \bar{\theta}_A \mid x_{A1}, \bar{\theta}_A \right] \cdot \delta_m
= \Pr \left[ \varepsilon_{A} \leq \bar{x}_{A1} - \bar{\theta}_A \mid x_{A1}, \bar{\theta}_A \right] \cdot \delta_m
= \frac{\bar{x}_{A1} - \bar{\theta}_A + \varepsilon}{2\varepsilon} \delta_m.
\] (A1)

From (A1), I get the following two equations:

\[
K = \frac{x_{A1P}^* - \theta_{AP}^* + \varepsilon}{2\varepsilon} \delta_P,
\] (A2)

\[
K = \frac{x_{A1O}^* - \theta_{AO}^* + \varepsilon}{2\varepsilon} \delta_O.
\] (A3)

Next, let’s think about the decisions of group 2 creditors. They know their own type ("pessimistic") but do not know the type of group 1 creditors. They can just conjecture the probability that the type of group 1 creditors is "pessimistic" as \( q \). They do not know the value of \( \bar{\theta}_A : \theta_{AP}^* \) or \( \bar{\theta}_A : \theta_{AO}^* \), either. Then the equation (1) becomes:

\[
K = \Pr \left[ \text{rollover is successful} \mid x_{A2}^* \right] \cdot \delta_P
= \left\{ \Pr \left[ \text{rollover is successful} \mid x_{A2}^* \right] + \Pr \left[ \text{rollover is successful} \mid \text{1’s are pessimistic} \mid x_{A2}^* \right] \right\} \cdot \delta_P
= q \times \Pr \left[ \theta_A \geq \theta_{AP}^* \mid x_{A2}^* \right] \cdot \delta_P + (1 - q) \times \Pr \left[ \theta_A \geq \theta_{AO}^* \mid x_{A2}^* \right] \cdot \delta_P
= q \times \frac{x_{A2}^* - \theta_{AP}^* + \varepsilon}{2\varepsilon} \delta_P + (1 - q) \times \frac{x_{A2}^* - \theta_{AO}^* + \varepsilon}{2\varepsilon} \delta_P.
\] (A4)

Lastly, let’s think about the critical threshold value of firm A’s fundamentals (i.e., switching fundamentals). The proportion of creditors who do not roll over the loans is expressed as follows:

\[
l_A (\theta_A) = \lambda \Pr \left[ x_{A1} \leq \bar{x}_{A1} \mid \theta_A \right] + (1 - \lambda) \Pr \left[ x_{A2} \leq x_{A2}^* \mid \theta_A \right]
= \lambda \Pr \left[ \theta_A + \varepsilon_{A} \leq \bar{x}_{A1} \mid \theta_A \right] + (1 - \lambda) \Pr \left[ \theta_A + \varepsilon_{A} \leq x_{A2}^* \mid \theta_A \right]
= \lambda \Pr \left[ \varepsilon_{A} \leq \bar{x}_{A1} - \theta_A \mid \theta_A \right] + (1 - \lambda) \Pr \left[ \varepsilon_{A} \leq x_{A2}^* - \theta_A \mid \theta_A \right]
= \lambda \frac{\bar{x}_{A1} - \theta_A + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x_{A2}^* - \theta_A + \varepsilon}{2\varepsilon}.
\]
The critical threshold value is determined by:

$$\tilde{\theta}_A = l_A (\tilde{\theta}_A) = \frac{\bar{x} A_1 - \tilde{\theta}_A + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x A_2 - \tilde{\theta}_A + \varepsilon}{2\varepsilon}. \quad (A5)$$

From equation (A5), I get the following two equations:

$$\theta^*_A P = \frac{\lambda x A_1 P - \theta^*_A P + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x A_2 - \theta^*_A P + \varepsilon}{2\varepsilon}, \quad (A6)$$

$$\theta^*_A O = \frac{\lambda x A_1 O - \theta^*_A O + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x A_2 - \theta^*_A O + \varepsilon}{2\varepsilon}. \quad (A7)$$

Solving equations (A2), (A3), (A4), (A6), and (A7), I get $x A_1 P$, $x A_1 O$, $x A_2$, $\theta^*_A P$, and $\theta^*_A O$. The unique equilibrium values of the switching fundamentals of firm A ($\theta^*_A P$ and $\theta^*_A O$) and the creditors’ switching private signals ($x A_1 P$, $x A_1 O$, and $x A_2$) are as follows:

$$\theta^*_A P = \frac{K}{\delta P} (1 - \Sigma_1),$$

$$\theta^*_A O = \frac{K}{\delta P} (1 - \Sigma_1 - \Sigma_2),$$

$$x^*_A 1 P = \frac{K}{\delta P} (1 - \Sigma_1 + \Sigma_3),$$

$$x^*_A 1 O = \frac{K}{\delta P} \left(1 - \Sigma_1 - \Sigma_2 + \frac{\delta P}{\delta O} \Sigma_3 \right),$$

$$x^*_A 2 = \frac{K}{\delta P} (1 - \Sigma_1 - (1 - q) \Sigma_2 + \Sigma_3),$$

where

$$\Sigma_1 = \frac{\lambda (1 - \lambda) (1 - q) (\delta O - \delta P)}{\delta O (1 + 2\varepsilon - \lambda)}, \quad \Sigma_2 = \frac{2\lambda \varepsilon (\delta O - \delta P)}{\delta O (1 + 2\varepsilon - \lambda)},$$

and

$$\Sigma_3 = \left(\frac{2K - \delta P}{K}\right)\varepsilon.$$

I also need to show that every creditor in each group strictly prefers not to roll over the loan (prefers to roll over the loan) if his private signal is less than (greater than) the switching private signal conditional on $\theta^*_A P$ and $\theta^*_A O$. Suppose that every other creditor follows the switching strategy. Then, an individual creditor in each group takes $\theta^*_A P$ and $\theta^*_A O$ as given. From equations (A2), (A3), and (A4), the present value of the expected payoff of rolling over the loan is strictly increasing in the switching private signals ($x^*_A 1 P$, $x^*_A 1 O$, and $x^*_A 2$) given $\theta^*_A P$ and $\theta^*_A O$. Therefore, for any private signal greater than the switching signal, the expected payoff of rolling over the loan is strictly greater than that of not rolling over. Thus, it is optimal for a creditor to follow the switching strategy, given that every other creditor follows the switching strategy.

**Derivation of $\theta_B P$ and $x_B$**

The proportion of creditors who do not roll over the loans conditional on $\theta_B$ is
expressed as follows:

\[ l_B (\theta_B) = \Pr \left[ x_B \leq x_B^* \mid \theta_B \right] = \Pr \left[ \theta_B + \varepsilon_B \leq x_B^* \mid \theta_B \right] = \Pr \left[ \varepsilon_B \leq x_B^* - \theta_B \mid \theta_B \right] = \frac{x_B - \theta_B + \varepsilon}{2\varepsilon}. \]

The critical threshold value of firm B’s fundamentals (i.e., switching fundamentals) is determined by:

\[ \theta_{BP}^* = l_B (\theta_{BP}^*) = \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon}. \quad (A8) \]

From the fact that creditors’ present value of the expected utility of rolling over the loans should be equal to the payoff from recalling the loan in the indifference condition, I get:

\[ K = \Pr \left[ \text{rollover is successful} \mid x_B^* \right] \cdot \delta_P = \Pr \left[ \theta_B \geq \theta_{BP}^* \mid x_B^* \right] \cdot \delta_P = \Pr \left[ x_B^* - \varepsilon_B \geq \theta_{BP}^* \mid x_B^* \right] \cdot \delta_P = \Pr \left[ \varepsilon_B \leq x_B^* - \theta_{BP}^* \mid x_B^* \right] \cdot \delta_P = \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon} \delta_P. \quad (A9) \]

From equations (A8) and (A9), I get the equilibrium strategy:

\[ \theta_{BP}^* = \frac{K}{\delta_P}, \quad x_B^* = \frac{K}{\delta_P} (2\varepsilon + 1) - \varepsilon. \]

**Derivation of \theta_{BO}^*, x_{B1}^*, \text{and } x_{B2}^*\)**

The proportion of creditors who do not roll over the loans conditional on \( \theta_B \) is expressed as follows:

\[ l_B (\theta_B) = \lambda \Pr \left[ x_{B1} \leq x_{B1}^* \mid \theta_B \right] + (1 - \lambda) \Pr \left[ x_{B2} \leq x_{B2}^* \mid \theta_B \right] = \lambda \Pr \left[ \theta_B + \varepsilon_{B1} \leq x_{B1}^* \mid \theta_B \right] + (1 - \lambda) \Pr \left[ \theta_B + \varepsilon_{B2} \leq x_{B2}^* \mid \theta_B \right] = \lambda \Pr \left[ \varepsilon_{B1} \leq x_{B1}^* - \theta_B \mid \theta_B \right] + (1 - \lambda) \Pr \left[ \varepsilon_{B2} \leq x_{B2}^* - \theta_B \mid \theta_B \right] = \lambda \frac{x_{B1}^* - \theta_B + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x_{B2}^* - \theta_B + \varepsilon}{2\varepsilon}. \]

The critical threshold value of firm B’s fundamentals (i.e., switching fundamentals) is determined by:

\[ \theta_{BO}^* = l_B (\theta_{BO}^*) = \lambda \frac{x_{B1}^* - \theta_{BO}^* + \varepsilon}{2\varepsilon} + (1 - \lambda) \frac{x_{B2}^* - \theta_{BO}^* + \varepsilon}{2\varepsilon}. \quad (A10) \]

From the fact that creditors’ present value of the expected utility of rolling over the loans should be equal to the payoff from recalling the loans in the indifference condition, I get the following equations for "optimistic" group 1 creditors and

\[ \text{ }}\]

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"pessimistic" group 2 creditors:

\[
K = \Pr[\text{rollover is successful} \mid x_{B1}^*] \cdot \delta_O \\
= \Pr[\theta_B \geq \theta_{BO}^* \mid x_{B1}^*] \cdot \delta_O \\
= \Pr[x_{B1}^* - \varepsilon_{B1} \geq \theta_{BO}^* \mid x_{B1}^*] \cdot \delta_O \\
= \Pr[\varepsilon_{B1} \leq x_{B1}^* - \theta_{BO}^* \mid x_{B1}^*] \cdot \delta_O \\
= \frac{x_{B1}^* - \theta_{BO}^* + \varepsilon}{2\varepsilon} \delta_O,
\]

(A11)

and

\[
K = \Pr[\text{rollover is successful} \mid x_{B2}^*] \cdot \delta_P \\
= \Pr[\theta_B \geq \theta_{BO}^* \mid x_{B2}^*] \cdot \delta_P \\
= \Pr[x_{B2}^* - \varepsilon_{B2} \geq \theta_{BO}^* \mid x_{B2}^*] \cdot \delta_P \\
= \Pr[\varepsilon_{B2} \leq x_{B2}^* - \theta_{BO}^* \mid x_{B2}^*] \cdot \delta_P \\
= \frac{x_{B2}^* - \theta_{BO}^* + \varepsilon}{2\varepsilon} \delta_P.
\]

(A12)

From equations (A10), (A11), and (A12), I get the equilibrium strategy:

\[
\theta_{BO}^* = \frac{\lambda K}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P},
\]

\[
x_{B1}^* = \frac{K (\lambda + 2\varepsilon)}{\delta_O} + \frac{(1 - \lambda) K}{\delta_P} - \varepsilon,
\]

\[
x_{B2}^* = \frac{\lambda K}{\delta_O} + \frac{K (1 - \lambda + 2\varepsilon)}{\delta_P} - \varepsilon.
\]

REFERENCES


