A Model of the “It” Products in Fashion

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Abstract

One of the characteristics of fashion is unpredictability and apparent randomness of fashion hits. Another one is the strong influence of the fashion editor recommendations on consumer demand. This paper proposes an analytical model of fashion hits in the presence of competition and a fashion editor acting on behalf of high-type consumers, in which we consider fashion as a means used by consumers to signal belonging to the high class in a matching game. We show that consistently with the observed market phenomenon, in equilibrium, the editor randomizes between available products. Furthermore, this randomization is essential for the market for fashion to exist when low-type consumers’ valuation of meeting high-type consumers is high enough. Whenever the low-type consumer demand for a product is positive, an increase in price results in higher probability of the product being chosen by the editor but lower low-type consumer demand. We also show that in equilibrium, firms always price as to attract strictly positive demand from the low-type consumers. The equilibrium price and profits are non-monotone in the low-type consumer valuation, with the equilibrium profit first increasing and then decreasing.

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“Though this be madness, yet there is method in’t.”
- Shakespeare

1 Introduction

Fashion refers to the style of clothing, shoes, handbags and other accessories, etc., prevalent at a given time.\(^2\) As opposed to the utilitarian products, fashion products are bought mostly for their social value, i.e., for their propensity to impress on others their user’s desirability, such as taste, wealth or otherwise belonging to a popular social group. For example, Simmel (1957) states: “Fashion... satisfies people’s needs for group cohesion and the need for individual elevation from the society.” As such, fashionable products are examples of what is known as status goods: products whose purpose is to signal that their user belongs to the desirable “high class” (Veblen 1899, Blumer 1969).

The distinguishing feature of fashion among other status goods is the unpredictability of fashion hits. Numerous new designs come out of fashion houses every season, but only a few products turn out to be popular among consumers. These fashion hits appear to emerge randomly: no one seems to be able to predict who will create the next hit product. For example, in the late 1990’s, the term “it bag” was coined to describe this phenomenon in the handbag market.\(^3\) It refers to the bag that becomes a popular best-seller and is deemed a must-have product (often unavailable by the time it is recognized as such). The fashion handbag market starting from the late 1990’s is a good example illustrating unpredictability of fashion hits. There is a constant competition for the it bag among top fashion houses, including Fendi, Hermes, Louis Vuitton, Mulberry, Prada and YSL. In 1998, Fendi created the first bag referred to as the “it bag” by the popular press – the original Fendi Baguette. With such a successful debut, one could expect the same company would be able to build on its success and create the it bags of the next seasons. However, in the years that followed, the it bag of the season was made by Prada in 1999, Dior in 2000, Balenciaga in 2001, Luella in 2002, Louis Vuitton in 2003, and so

\(^1\)Hamlet (1604), Act 2, scene 2, 205-206.
\(^2\)E.g., see the Wikipedia entry at http://en.wikipedia.org/wiki/Fashion
\(^3\)E.g., see the Wikipedia entry at http://en.wikipedia.org/wiki/It_bag.
on (Rumbold 2007). Although all fashion houses want to create the it bag, not only they do not know whether their design will become the it bag or not, they don’t even seem to be able to do much to affect the choice. The whimsical opinion of fashion editors, such as one famously portrayed in the Hollywood movie “The Devil Wears Prada,” may be far more important for setting the trend than the firm’s promotional efforts. For example, Betts (2006) quotes Stuart Verves, the designer behind the “suddenly hot British brand Mulberry,” as saying “creating an ‘It’ bag is just dumb luck. ... You have to wait for your time” to support her point that “the creative coup is often more the result of serendipity than science.”

This observation that randomness seems to be at the heart of fashion motivates our research. If one is to make predictions of and recommendations for the optimal firms’ behavior in the fashion industry, one needs to understand the rules that govern the randomness of the fashion hits. This paper analyzes the fashion hit selection, and shows that it is intrinsically random, but the probability of a product becoming the fashion hit is in equilibrium uniquely defined. It then builds on this result to make predictions of and recommendations for the firms’ optimal profit-maximizing strategy in a competitive market.

1.1 The Fashion Industry

Products of the fashion industry are usually classified by season, especially for high fashion. Fashion houses release their new designs at Fashion Weeks, usually six months ahead of the season. For example, Fall/Winter collections are usually showcased in these events between January and April. Fashion editors and journalists would attend these events and gather information about new designer collections. Then they will screen these collections and selectively inform the consumers about what’s going to be “hot” in the upcoming season through fashion media, such as fashion magazines, fashion blogs and TV programs. Consumers acquire this information and decide what to buy when the products actually arrive at the market. A particular product that catches on among consumers and become a symbol of “in” is often referred to as the “it” product.

Prices of fashion products are usually decided at a very early stage and before the “it”
product is determined. For example, according to Eileen Balaban-Eisenberg, the executive V.P. of the Connaught Group, New York, fashion designers make the product price information available for the key influencers in the fashion world even before the fashion shows at which the products are introduced, although this price is not yet made available for the general public at the time.\footnote{From the personal interview of Eileen Balaban-Eisenberg with the authors.}

Note that some status goods, such as diamonds, luxury watches, cars, and certain handbags are well known to be status goods and owning any sufficiently expensive product itself is enough for signalling. For example, having a Lamborghini may be all that is needed to impress one’s wealth on others. For fashion products, however, what matters is whether one has the right product. The recognition of a person as “in” crucially depends on the specific product she uses. Not everyone has enough information about what is the right product. The ability to acquire timely and accurate information is an important characteristic that differentiates one consumer from another in the fashion market. Those with good access to fashion information know what is in trend and buy accordingly. Those with poor ability to acquire information can only guess at what to buy. Of course, they could also wait to find out what was fashionable, but buying late is usually synonymous with buying nothing as far as the fashion statement is concerned, since popular designs often become sold-out for several months.\footnote{Reasonably attractive fashion hits of one season often do enjoy popularity for several years, but no longer as a fashion statement.} Once they are widely available and recognized, the popularity often spells doom for the prestige.\footnote{As, for example, Yogi Berra explained about restaurant Ruggeri’s, “Nobody goes there anymore. It’s too crowded.”}

To give a specific example, Chow (2009) depicts the following story. In 2009, the Swedish fashion brand H&M in cooperation with designer Jimmy Choo developed a special line of products under the name “Jimmy Choo for H&M.” With the fashion news columnist buzz that the line is going to be a hit, consumers “in the know” lined up in front of the store throughout the night before the scheduled release on November 14th, 2009, so that they could be the first ones to get in. By 1 pm that day, most of the special collection products were no longer available. As illustrated by this example, good access to fashion information is essential for consumers to
get the “in” product before it’s gone.

1.2 The Role of the Fashion Editor

Fashion editors are very important agents in the fashion industry, since their choices and recommendations have considerable impact on the trend of fashion. In fact, some industry insiders argue that fashion editors are the single most important influencer of fashion.\(^7\) But their role have been largely ignored in the academic literature.

In some instances, it may also be that fashion is decided by a relatively small group of “high-class” consumers such as movie stars or other celebrities, a collective chatter of connected “fashion mavens,” or trend-setters at elite clubs. Hash Puppies popularized by a few enthusiasts (and the actor/fashion designer Isaac Mizrahi) in 1994 (Gladwell 2000) and an up-to-6-month shortage of UGG boots in late 2003 after a number of actresses were seen wearing them (Grant 2003) are some examples of the role of the narrow set of opinion leaders. In what follows, by the editor, we mean this force that serves to coordinate the behavior of the “in” consumers and facilitates the creation of fashion hits, be it a physical most-influential editor such as the editor of Vogue magazine to whom members of “high society” have easier access, a collective of less known fashion bloggers whose blogs are so numerous that they can be navigated through only by the “in-the-know” consumers, or the coordinating events such as parties attended by celebrities, the news from which spreads through word-of-mouth to the high-social-status consumers first and then to the rest.

The interaction between firms, editors, and consumers in the fashion industry is in the center of our research. Our model shows how the randomness of fashion hits comes as a result of high-status consumer coordination and the optimal response to the low-type consumer behavior. As we have stated before, the first objective of this model is to understand the driving forces and the rules of randomness of fashion hits. The second objective is to predict and make recommendations for the optimal competitive firm strategy in such a market.

\(^7\)As Eileen Balaban-Eisenberg puts it, “fashion editors do not predict trends, they dictate them.”
1.3 Summary of Results

We develop a model of the editor and two firms competing in a fashion market with two consumer types: the high and the low type. The high-type consumers are the “socially desired” type, and all consumers would prefer being recognized as of the high type than as of the low type. Consumer types are their private knowledge and are not directly observable by other consumers. Other consumers however form expectations of a given consumer type based on the product she uses. The game starts with each firm introducing one product, after which the editor acting on behalf of the high-type consumers makes a recommendation to consumers of whether to buy a product and if so, which one. Only the high type consumers know the editor’s pick. In the next stage, consumers make product purchase decision. Finally, consumers enter the social interaction stage, which we call the “matching game”, in which they are randomly matched into pairs with other consumers who made the same product choice. Keeping in line with the idea that fashion is used mainly for social reasons, we assume that consumer derive utility of a product only through its effect on the type of the person they will interact with in the matching game.

We find that in equilibrium, the editor will always randomize between the two products. In other words, in equilibrium, the presence of the editor always leads to randomness of fashion hits. Although fashion hits are random, the probability of each product being selected as the “it” is uniquely defined in the equilibrium as a function of the product’s price. Thus, unlike in the case without the editor, the expected sales are predictable and the expected profit functions can be uniquely derived.

Another feature of this model is that market for fashion always exists when there is an editor. Again, this contrasts the no editor case, where market could only exist if the valuation of the low-type consumers for being recognized as “in” is lower than that of the high-type consumers.

Third, we find that the probability that a product will be chosen increases in own price if and only if the equilibrium demand for it from low-type segment is positive, while the equilibrium demand from the low type consumers always decreases in price. The rationale for this result is that when the editor decides which product to pick, she has to trade off the following two
implications of a higher price. One is the direct cost due to the higher price. The other is the possibly reduced demand from the low types, which would increase the signalling value of the product. A priori, it is not clear which effect dominates the other when the second effect exists (i.e., when the demand from low-type consumers is positive). This result indicates that the indirect utility is a more important consideration than the direct one (price) for the editor.

Fourth, we find that in a pure strategy equilibrium, each product is priced so as to receive strictly positive expected demand from the low-type consumers. Thus, in view of the previous result, in the neighborhood of a pure strategy equilibrium price, the editor appears to prefer high-priced products: even though the editor considers the direct effect of a higher price as a negative factor in choosing a product, she will end-up selecting the product with higher probability if its price is increased from the equilibrium. We also find that the editor prefers the prices to be higher than their optimal level for the firms.

Fifth, we find that as a function of the low-type consumer valuation of being “in”, the equilibrium prices and profits are not monotonic. In particular, as this valuation increases from zero to infinity, the equilibrium price first increases, then decreases, then increases again, and finally decreases. On the other hand, even though the total demand increases as the valuation increases, the profit first increases and then decreases. In other words, there is a level of low-type valuation of matching with the high-type consumers which is optimal for the producers.

Finally, when compared with the monopoly case, we find that competition may increase price. This happens when low-type consumers’ valuation for being recognized as “in” is above that of the high-type consumers, but is not too high.

The rest of the paper is organized as follows. Section 2 reviews the literature related to fashion and status goods. Section 3 fully specifies the model. Section 4 presents an analysis of consumer purchase decision and the editor’s choice of recommendation. Section 5 discusses the full equilibrium of the game and provides insights into the market dynamics. Section 6 concludes.

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8Depending on the parameter values, the equilibrium will be either in pure or mixed strategies.
2 Literature Review

The economic analysis of fashion and fads considered two possible sources of seemingly random consumer coordination on a particular product within a category. One is the information cascade resulting from sequential consumer choice under incomplete information about the product. For example, Bikhchandani, Hirshleifer and Welch (1992) argue that the convergence behavior we observe in fashion and fads can be explained through an informational cascade model. In their model, individuals act in sequence and each individual makes his choice based on the action of all the people acting before her and the private signal she receives. In the equilibrium, it is optimal for later consumers to follow the behavior of the very early ones and ignore their own information. This leads to the convergence of consumer behavior.

Another stream of literature emphasizes the role of fashion as a signalling device. Karni and Schmeidler (1990) study the variation in the demand for fashion products from a purely consumer perspective without incorporating firms in their model. In their model, there are three colors of a product and two types of consumers. Consumers of one type prefer more consumers of the same type and fewer of the other type to use the same color as them, while consumers of the other type prefer more of both types of consumers to use the same color as them. In the equilibrium, there is a dynamic variation in demand for different colors, which mimics the dynamics of fashion cycles. Pesendorfer (1995) proposes a dynamic game between a monopoly firm and consumers to explain the occurrence of fashion cycles. Consumers are of two types and everyone prefers to be matched with the desired type of consumers. The game is played for infinitely many periods and in each period, consumers’ payoffs are determined by the type of people they are matched with, which then depends on the fashion design they use. Fashion cycle arises in the equilibrium. As Pesendorfer notes, there is an inherent issue in modeling competition in a market for signalling devices. In competition in most markets, if products are not differentiated, the result is marginal cost pricing, the sales going to the product with the lowest marginal cost. In a market for signalling devices, a price reduction may rob the product of its value. Therefore, competition may not result in price declining to marginal costs and any
combination of market shares may constitute an equilibrium.\footnote{This indeterminism of profit expectations is also implied by the models in Bagwell and Berheim (1996) and Becker (1991).}

Our paper adopts the second approach of looking at fashion as a signalling device and considers the role of fashion editors, or more broadly, of the coordination of high-type consumers, in the fashion industry. As we will see, considering the fashion editor behavior allows us to model another important aspect of fashion industry – competition – in which expected profits are uniquely defined in equilibrium. This model contributes to our understanding of fashion markets by proposing a theory to both explain the randomness of fashion hits, and predict the likelihood of a product becoming a fashion hit.\footnote{For example, Yoganarasimhan (2008) documents the importance of randomness in fashion, but assumes that fashion hits are exogenously random.}

\section{Model Setup}

The market consists of two firms offering one product each, the editor, and a unit mass of consumers of two types: the high-type consumer segment has mass $\alpha$ and the low-type has mass $1-\alpha$. Following product purchase decisions, consumers engage in social interaction with payoffs determined by the type of consumers they interact with. The high-type consumers are, by definition, more desirable. As the consumer type is not directly observed, consumers try to signal their type through the product choice. This signalling value of the product is the only utility consumers derive from the products. This captures the notion that the fashion is social device and its utility is derived from social interactions.

Specifically, following Pesendorfer (1995), we model the social interaction through a “matching game,” in which consumers are randomly paired up with other consumes with the same product use. As in that model, we assume consumers of the same type are to be homogenous, so that the payoff of a match between consumers of types $j$ and $k$ depends on $j$ and $k$ only. We then normalize any consumer’s payoff from matching with a low-type consumer to 0, and payoff of the high-type matching with a high-type consumer to 1. We denote the remaining parameter – the payoff of the low type consumer from pairing up with a low type consumer by
V. Consumer’s objective in product choice is to maximize the expected payoff in the matching game net of the product’s price.

The editor’s objective function is to maximize the expected payoff of the high-type consumers, and we assume that the high types have a higher ability of acquiring information about the editor’s choice. Both of these assumptions would naturally be true if the editor was representing the collective decision making by the high-type consumers or their subset, but even when the editor is a physical editor of a fashion magazine, it would normally be elected from and supported by the high-type consumers.\textsuperscript{11} Alternatively, one can think about the ability of acquiring information being correlated with social status. For tractability and to be consistent with the in-type homogeneity assumption above, we further assume that all high-type consumers perfectly observe the editor’s pick before the purchase decision, but the low-type consumers acquire no information about it until after the purchase decision stage. However, we assume that right before the matching game, low-type consumers also find out the editor’s choice, but they can no longer purchase. They could, however, possibly discard an item and not use it in the matching game even if they have purchased it. This assumption is not essential for the main results.

The firms maximize their individual profits and have a constant and equal marginal product cost, which we normalize to 0. The game sequence and decisions are as follows:

1. Firms 1 and 2 simultaneously introduce products and set prices.
2. Editor picks recommendation: product 1, product 2, or neither. The high-type consumers learn the editor’s choice, while the low-type consumers do not.
3. Consumers make purchase decisions.
4. All consumers learn the editor’s recommendation, each consumer decides which product to use, if any, out of the products purchased. The “matching game” is then played out.

To fully define the payoffs in the matching game, we assume that choosing a product not chosen by anybody else results in zero payoff.\textsuperscript{12} We use subgame-perfect Nash equilibrium as the

\textsuperscript{11}One possible explanation for this setup is that the high types are the subscribers of fashion magazine. So the editor has to work for the welfare of the high types to keep these customers. Another explanation is that the “editor” refers to a small group of high type consumers, so they work for their own welfare.

\textsuperscript{12}This assumption is not essential: alternatively, and with no change in equilibrium results for all but a zero mass of consumers, we could define this payoff to be any value.
solution concept, and solve the model by backward induction. Since the editor’s objective is to maximize the high-type consumer utility, we will only focus on equilibria in which the high types find it optimal to completely follow the editor’s recommendation, i.e., we assume that if the editor picks a product, all high-type consumers buy it, and if she picks neither, no high-type consumers buy either product. This is a natural assumption since the editor maximizes the high-type consumer utility and therefore it is always equilibrium for the high-type consumers to follow the editor’s pick.\footnote{An equilibrium in which high-type consumers ignore the editor’s recommendation also always exists, since if all consumers ignore any recommendation, there is no reason for any consumer to follow any recommendation. But considering such an equilibrium amounts to ignoring the role of the editor.}

### 4 Consumer Choice and the Editor’s Strategy

We devote special attention to the consumer and editor strategies rather than just to the equilibrium predictions of the full game, because fully considering all possibilities of the subgame starting from the editor’s choice would allow us not only to predict the equilibrium outcome of the full game, but also to analyze the profit-maximizing strategy of a firm conditional on its expectation of the other firm’s choices.

The consumer benefit of using a product is determined by the probability that the consumer will be matched with a high-type consumer. Therefore, the utility of purchasing Product \( k \) for a high- and low-type consumer is given by

\[
U_h(k) = q_k - p_k \quad \text{and} \quad U_l(k) = q_k V - p_k,
\]

respectively, where \( q_k \) is the fraction of high-type consumers among users of product \( k \). To keep notation parsimonious, we will refer to the option of “neither” product as product \( k = 0 \) with the associated price of this option being \( p_0 = 0 \). Further, denote the low-type consumer demand for product \( k \) by \( x_k \).

Note that the editor can ensure high-type consumer payoff of \( \alpha \) by recommending “neither,” which would be optimal for all high-type consumers to follow. Therefore, it is strictly suboptimal for the editor to recommend a product with price above \( 1 - \alpha \). Given that the low-type consumers
can expect this part of the editor’s strategy, they will have no demand for such a product either. Thus, price above $1 - \alpha$ guarantees zero sales and profits. It will be soon clear that each firm
can guarantee positive sales by pricing sufficiently low, and therefore, pricing above $1 - \alpha$ is a
strictly dominated strategy. It will imply that pricing at 0 is also a strictly dominated strategy.
Therefore, we restrict our analysis to the consumer and editor’s choices under the following
condition:

$$0 < p_2 \leq p_1 < 1 - \alpha,$$

assuming without loss of generality that $p_2 \leq p_1$.

Solving the game by backward induction, let us consider first consumer product use in
Stage 4. If the editor recommended product $k \neq 0$, the users of product $k$ will consist of all
high-type consumers and those low-type consumers who purchased it. The low-type consumers
who bought product $j = 3 - k$ will chose not to use it. Therefore $q_k = \frac{\alpha}{\alpha + x_k}$ and $q_0 = 0$. The
equilibrium value of $q_j$ is either not defined (if the low-type consumers who purchased product $j$
all decide not use the product) or is zero (if some low-type consumers bought product $j$ and at
least one of them decides to use it), but the matching game payoff from using product $j$ is zero
in either case. If the editor recommended $k = 0$, i.e., not to buy either product, all consumers
will end up not using either product in the matching game. This would lead to $q_1 = q_2 = 0$ and
$q_0 = \alpha$.

In Stage 3, a high-type consumer follows editor’s recommendation, but a low-type consumer
have to make her decision based on her expectation of the editor’s strategy and her expectation
of the proportion of other low-type consumers buying each product. She can choose product 1,
product 2, both or neither.

A low-type consumer purchase of product $k \neq 0$ gives her the expected matching utility of
$q_k V$ when the editor recommended product $k$ and 0 if the editor recommended product $3 - k$.
This incremental utility from the product choice does not depend on whether she also decides
to purchase product $3 - j$ or not, which means that low-type consumer purchases of the two
products can be treated as two separate decisions. Specifically, the consumer will purchase
one unit of product $k$, $k = 1, 2$, if and only if $U_t(k) \geq U_t(0)$. However, the equilibrium low-
type consumer demands for the two products end up being inter-related because the editor’s decision on whether to recommend product \( k \) and its expectation by the low-type consumers is dependent both on the editor’s trade-off in recommending product \( k \) and \( j = 3 - k \). Note that \( U_l(k) \) decreases in the mass of low-type consumers who decide to buy product \( k \). Given the low-type consumer expectation of the probability with which product \( k \) is selected by the editor, the low-type consumer demand for either product will therefore be determined by equating \( U_l(k) = U_l(0) \) whenever this equation leads to an internal solution.

In Stage 2, the editor needs to form her expectation of the low-type consumer behavior in the following period, given \( p_1 \) and \( p_2 \), and accordingly choose her optimal recommendation. Since the editor’s choice depends on her expectation of the low-type consumer decisions and low-type consumer decisions depend on their expectations of the editor’s choice, the equilibrium follows from the simultaneous solution to editor’s and the low-type consumers’ equilibrium conditions. A-priori, the editor’s strategy must be one of the following: (1) follow a pure strategy and choose one of the options; (2) randomize between two options, e.g., picking product 1 and product 2 or between product 1 and neither, and (3) randomize between all three options. We will show that given prices satisfying Equation (2), choosing “neither” only or randomizing between one of the products and “neither” is never optimal. Therefore, the equilibrium conditions on the editor’s choice will come from indifference between the two products or all the three options, or choosing one product only.

In equilibrium, it can not be that all low-type consumers buy either of the products. This is because if all the low types buy a product, then the net of price payoff for the high types buying that product must be smaller than \( \alpha \) and hence the editor has the incentive to stop picking that product and choose neither instead.

Furthermore, we will show that whenever the low-type consumer demand for each of the two products is positive, it is optimal for the editor to randomize across all the three options, and the equilibrium pricing strategy is such that in expectation, each product receives positive demand from the low-type consumers, although when the equilibrium pricing strategy is mixed, the optimal strategy for the editor is to randomize between the two products only and for the
Proposition 1. Given two prices satisfying Equation (2), we have

1. ("Low prices") If \( \alpha(1 - V)(p_1 + p_2) + (2 - V)p_1p_2 < \alpha^2V \), the editor randomizes between all three options and low-type consumer demand is positive for both products.

2. ("Medium prices") If \( p_2(1 - V + p_1) + (p_1 - V)(1 - p_1) < 0 \) but \( \alpha(1 - V)(p_1 + p_2) + (2 - V)p_1p_2 > \alpha^2V \), the editor randomizes between Products 1 and 2 and low-type consumer demand is positive for both products;

3. ("High prices") Otherwise, if \( p_2(1 - V + p_1) + (p_1 - V)(1 - p_1) > 0 \) but \( p_1 \leq 1 - p_2/V + p_2 \), the editor randomizes between Products 1 and 2 and the low-type consumer demand is positive only for Product 2, and

4. ("Very high prices") If \( p_1 > 1 - p_2/V + p_2 \), the editor will choose Product 2 with probability 1 and Product 1 receives no demand;

For \( p_1 > p_2 \), the editor’s recommendation probabilities and the low-type consumer demands for the two products in the four areas above are stated in Table 1. For \( p_1 = p_2 \), the equilibrium is different from what’s stated in Table 1 only in Area 3. In that case, any subgame in which the editor picks the two products with probability \( \delta_1 \) and \( 1 - \delta_1 \), where \( \delta_1 \in [p_1/V, 1 - p_1/V] \) is a possible equilibrium outcome, and the low-type consumer demand for each product is zero.

Proof. See the Appendix. \( \square \)

Although there are multiple equilibria for \( p_1 = p_2 \) in the high price area, it’s most natural to assume that the editor will choose both products with equal probability. Hereafter, we’ll make this assumption whenever needed in solving the full game.

<table>
<thead>
<tr>
<th>Area 1</th>
<th>Editor’s strategy</th>
<th>Low-type consumer demand</th>
</tr>
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<tbody>
<tr>
<td>( \delta_k = \frac{p_k}{V(p_k + \alpha)} )</td>
<td>( x_k = \frac{\alpha(1 - \alpha - p_k)}{\alpha + p_k} )</td>
<td></td>
</tr>
<tr>
<td>Area 2</td>
<td>( \delta_k = \frac{1}{2} + \frac{(2 - V)(p_k - p_j)}{p_1 + p_2 + \sqrt{(p_1 - p_2)^2(V - 1)^2 + 4p_1p_2}} )</td>
<td>( x_k = \frac{\alpha V}{p_k} - \alpha + \frac{\alpha V(2 - V)(p_k - p_j)}{p_k(p_1 + p_2 + \sqrt{(p_1 - p_2)^2(V - 1)^2 + 4p_1p_2})} )</td>
</tr>
<tr>
<td>Area 3</td>
<td>( \delta_1 = \frac{V - p_2 - V(p_1 - p_2)}{V(1 - p_1 + p_2)} ); ( \delta_2 = \frac{p_2}{V(1 - p_1 + p_2)} )</td>
<td>( x_1 = 0; \ x_2 = \frac{\alpha(p_1 - p_2)}{1 - p_1 + p_2} )</td>
</tr>
<tr>
<td>Area 4</td>
<td>( \delta_1 = 0; \ \delta_2 = 1 )</td>
<td>( x_1 = 0; \ x_2 = \max{0, \frac{\alpha(V - p_2)}{p_2}} )</td>
</tr>
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Table 1: Editor’s Strategy and Low-Type Consumer Demand for \( p_1 > p_2 \) \((k = 1, 2; j = 3 - k)\).
Some areas may be empty for some $V$ or $\alpha$. Area 4 is not empty only when $V < 1 - \alpha$. The other three areas are not empty for $V < 2 - 2\alpha$. For $V \geq 2 - 2\alpha$, the medium- and high-price regions disappear as well. It is then immediately apparent that for $V$ above this cut-off, the full equilibrium of the game must involve the editor selecting “neither” with positive probability.

Note that Area 1 is the only one where the editor will select “neither” with positive probability. This is because the high-type consumers can only get the payoff of $\alpha$ if the editor picks neither. She must select a product to make the high-type consumers get higher payoff. When prices are low enough, no randomizing between the two products would prevent the low-type consumers from buying, and the editor ends up choosing neither product with a positive probability.

Figure 1 illustrates the areas in Proposition 1 and how they change with $V$.

Note that in Area 1, the editor’s probability of choosing neither increases in $\alpha$. This is because when the fraction of the high type consumers becomes larger, the utility from purely random match increases, making the neither option more attractive. The probability of picking neither also increases in $V$. This says as the low types’ valuation for social interactions increases relative to that of the high type consumers, it becomes more and more difficult to create a fashion hit without letting the low type consumers buy. In fact, as $V$ goes to infinity, the probability that the editor recommends any product tends to zero. Also note that as a product’s price decreases to zero, the product fully penetrates the low-type consumer segment even as its probability of being chosen by the editor tends to zero. Examining the editor’s choice probabilities in Table 1, we further obtain the following result.

**Corollary 1.** An increase in a product’s price leads to the editor picking it with higher probability but fewer low-type consumers buying it in all regions except the higher priced product in Area 3 and 4 above. In Area 3 above, an increase in the higher-priced product leads to lower probability of it being chosen by the editor.

*Proof.* Immediately follows from Table 1. □
The intuition for this result is as follows. When one of the prices increases, the editor is trading off between the cost and utility of matching. Higher price reduces the utility of the high type consumers directly and hence makes the editor select that product less frequently. However, an increase in price also inhibits the low type from buying, which enhances the high types’ utility indirectly. This encourages the editor to select that product more frequently. The above corollary states that whenever there is such a tradeoff, the gain from higher matching utility always dominates the cost effect, resulting in the editor picking that product with higher probability, but so that the demand from low types decreases. Of course, for the higher-priced item in the high-priced area this reasoning does not work because in that area, the higher-priced product receives zero demand from the low-type consumers. So the above-discussed tradeoff is nonexistent and the only consideration for the editor is the direct cost effect of the price, which leads her to select the higher-priced product less frequently if its price increases.

Before proceeding to examining the optimal firms’ strategy, note that the editor’s preferred price outcome is $p_1 = p_2 = V/2$ when $V < 2 - 2\alpha$ and the editor is indifferent between any prices otherwise. At $p_1 = p_2 = V/2 < 1 - \alpha$, the editor will only pick the two products and randomize between them with equal probability. The low types buy neither product, but they are just indifferent between buying and not buying. The above prices are the best for the high-type consumers because they prefer to have the cheapest “prestige” product that only the high type consumers will buy.

The socially optimal outcome is $p_1 = p_2 = V/2$ for $V < 1$ and $p_1 = p_2 = 0$ for $V > 1$. Intuitively, when low type consumers’ matching utility is relatively low compared with that of the high types, they should be separated from the high-type consumers to ensure that the high type consumers realize the highest matching utility. When the opposite is true, then the social optimal outcome should ensure that the low types are matched with high-type consumers as often as possible. Thus, for $V < 1$, the editor’s preference is the same as the socially optimal outcome, but for $V > 1$, the editor prefers higher prices than socially optimal.
Table 2: Profit Functions Given Prices $p_1 > p_2$ ($k = 1, 2; j = 3 - k$).

| Area 1 | $\pi_k = \frac{\alpha p_k (V + P_k - V P_k - a V)}{V (a + p_k)}$ |
| Area 2 | $\pi_k = \alpha (\frac{V}{2} - \frac{p_k}{2}) + \frac{(2 - V)(p_k - p_j)(p_k + V)}{p_1 + p_2 + \sqrt{(p_1 - p_2)^2 + 4 p_1 p_2}}$ |
| Area 3 | $\pi_1 = \frac{\alpha p_1 (V + V p_2 - p_2 - V p_1)}{V (1 - p_1 + p_2)}$, $\pi_2 = \frac{\alpha p_2 (p_2 + V p_1 - V p_2)}{V (1 - p_1 + p_2)}$ |
| Area 4 | $\pi_1 = 0$, $\pi_2 = \max \{\alpha p_2, \alpha V\}$ |

5 Pricing Strategy and the Equilibrium of the Full Game

Table 2 reports firms’ profits given the prices and the equilibrium editor’s strategy and low-type consumer demand. These profit functions are concave in own price in the medium and low price areas. In the high price area, the profit function is concave in own price when it is above the competitor’s price. Furthermore, the profit function changes discontinuously when the price decreases from just above to just below the competitor’s price in the high price region; has a downward kink when the price increases from the low to medium price region, and has an upward kink when crossing from medium to high price region. When $V < 1$, responding to any price with the optimal price that puts prices in the high-price region is always better than responding with the optimal price that puts prices in the medium price region. When $V > 1$, the opposite is true.

An interesting consequence of the equations in Proposition 1 is that the tradeoff between price and demand for the profit maximization of a firm in this model is not necessarily the usual negative relationship between price and demand. In this model, lower price may lead to a lower total demand. This is because, as Proposition 1 states, in all but one region, higher price increases the product’s probability of being chosen although decreasing the demand from the low-type consumers. This is a result of fashion being a signaling device and its value endogenously coming from what it means in social interaction. When low type consumers are buying more, signaling becomes more difficult and hence the value of signaling decreases for the high type consumers. Hence there is a tension between the demand from high types and that from low types, and the overall effect on demand is a priori not clear. Theoretically, it
is possible that higher price may create higher demand. For example, in the low-price region, when \( V < \alpha \), total demand would increase with the product’s own price.

If there is a negative relationship between price and demand, then the firm has to tradeoff between the two. Since the demand in our model is continuous, lowering or raising price doesn’t generate a huge gain or loss of demand. In this case, the firm faces the nontrivial choice between charging a high price to create the premium image (or, in the model, increase the probability that the product will end up having a premium image) and charging a lower price to generate more sales. Note that this tradeoff if not present in the absence of the editor as the absence of premium image would necessarily also imply absence of consumer demand from either type.

5.1 Equilibrium Price

The shape of the profit functions described at the beginning of Section 5 implies that depending on the slope of the profit functions, a pure-strategy equilibrium price may either be in the low-price region, on the boundary of low and medium price region or in the medium price region. As we will show below, when \( V \) decreases from infinity to 1, the equilibrium is indeed in pure strategies and traverses the above cases respectively.

Given the above-discussed globally-optimal response to the optimal price of the medium price region, for \( V < 1 \), the equilibrium must be in mixed strategies that at least sometimes put prices in the high price region. A pure-strategy equilibrium cannot be in the high price region, because in that region, slightly undercutting the competitor’s price is always better than pricing equally with the competitor. As we will argue later, the mixed strategy equilibrium in this case places prices in the high-price region. But first, we summarize the pure price strategy equilibria in the following proposition.

**Proposition 2.** Let \( V_1 = 2 + \frac{a}{2} - \frac{\sqrt{\alpha^2 + 16\alpha}}{2} \) and \( V_2 = 2 + \frac{a}{2} - \frac{\sqrt{\alpha^2 + 8\alpha}}{2} \) and assume \( V > 1 \). Then the symmetric equilibrium prices and profits are reported in Table 3, and the corresponding equilibrium editor’s strategy (\( \delta_k \) for \( k = 1, 2 \)) and the low-type consumer demand is reported in Table 4. Furthermore, the equilibrium is unique unless \( V \in (\max\{1, V_1\}, \min\{V_2, 2 - 2\alpha\}) \), in which case there are also asymmetric equilibria. When \( V < 1 \), there are no pure strategy equilibria. The equilibrium price decreases in \( V \) and equilibrium low-type consumer demand increases in \( V \) for
Table 3: Equilibrium Prices and Profits ($k = 1, 2, j = 3 - k$).

<table>
<thead>
<tr>
<th>Region of $V$</th>
<th>Prices</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $[1, V_1]$</td>
<td>$p_k = \frac{V(V - V)}{2 + V}$</td>
<td>$\pi_k = \frac{\alpha V^2}{2 + V}$</td>
</tr>
<tr>
<td>2. $(\max{1, V_1}, \min{V_2, 2 - 2\alpha})$</td>
<td>$p_k = \frac{\alpha V}{2 - V}$</td>
<td>$\pi_k = \frac{\alpha V^2}{2(2 - V)}$</td>
</tr>
<tr>
<td>3. $[2 - 2\alpha, V_2]$</td>
<td>$p_k = 1 - \alpha$</td>
<td>$\pi_k = \frac{\alpha(1 - \alpha)^2}{V}$</td>
</tr>
<tr>
<td>4. $[V_2, \infty]$</td>
<td>$p_k = \sqrt{\frac{\alpha V - \alpha}{V - 1} - \alpha}$</td>
<td>$\pi_k = \alpha + \alpha^2 - 2\alpha(\alpha + \sqrt{\alpha(V - 1)(V - \alpha)})$</td>
</tr>
</tbody>
</table>

Table 4: Equilibrium Editor’s Choice Probabilities $\delta_k$ and Low-type Consumer Demand $x_k$ ($k = 1, 2, j = 3 - k$).

<table>
<thead>
<tr>
<th>Region of $V$</th>
<th>Editor’s Strategy</th>
<th>Low-Type Consumer Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $[1, V_1]$</td>
<td>$\delta_k = 1/2$</td>
<td>$x_k = 1 - \alpha - V/2$</td>
</tr>
<tr>
<td>2. $(\max{1, V_1}, \min{V_2, 2 - 2\alpha})$</td>
<td>$\delta_k = 1/2$</td>
<td>$x_k = 1 - \alpha - V/2$</td>
</tr>
<tr>
<td>3. $[2 - 2\alpha, V_2]$</td>
<td>$\delta_k = \frac{1 - \alpha}{V}$</td>
<td>$x_k = 0$</td>
</tr>
<tr>
<td>4. $[V_2, \infty]$</td>
<td>$\delta_k = \frac{1}{V} (1 - \sqrt{\alpha(V - 1)/V - \alpha})$</td>
<td>$x_k = \sqrt{\alpha V - \alpha - \alpha}$</td>
</tr>
</tbody>
</table>

$V \in ([1, V_1] \cup [V_2, \infty]$ and the equilibrium price increases in $V$ while the low-type consumer demand decreases in $V$ in the symmetric equilibrium when $V \in (\max\{1, V_1\}, \min\{V_2, 2 - 2\alpha\})$.

Restricting attention to the symmetric equilibrium, the equilibrium profit increases with $2 - \sqrt{2\alpha}$ and then decreases, thus achieving the highest value for $V \in (\max\{1, V_1\}, \min\{V_2, 2 - 2\alpha\})$.

Proof. See the Appendix.  

Figures 2 and 3 illustrate the equilibrium prices and profits, respectively, as a function of $V$ when $\alpha = 1/4$. The graphs for other parameter values are similar.

The equilibrium in Region 1 and 2 corresponds to the medium-price area in Proposition 1, in which case “neither” is selected with positive probability by the editor. The equilibrium in Region 3 lies on the boundary between the low-price and medium-price region, and that in Region 4 corresponds to the low-price region. In the latter two areas, the editor always picks
one of the two products and the low type consumers by both. Note that when \( \alpha > 1/3 \), we have \( V_1 < 1 \) and therefore Region 1 does not exist. Region 3 also disappears for \( \alpha > 1/3 \).

To intuitively understand the driving forces leading to the equilibrium stated by the above proposition, one can observe that when \( V \) is high enough (Region 4), the medium and high price regions do not exist and therefore the equilibrium is derived from the first-order conditions on the profits in Area 1. In Region 3, the solution to the first-order conditions comes up to the binding constraint \( 1 - \alpha \), above which the editor would prefer to give up on trying to separate the high- and low- type consumers and thus the price becomes \( 1 - \alpha \).

In Region 2, the medium price boundary crosses the boundary \( p_1 = p_2 = 1 - \alpha \) and firm \( k \) incentives to decrease price \( p_k \) from \( p_1 = p_2 = 1 - \alpha \) change. Specifically, the first-order conditions of the medium price range suggest decreasing price while the first-order conditions of the low-price region suggest increasing price. Therefore, the equilibrium prices lie on the boundary between these two cases. As one would expect in the games with such downward kink in the profit function, there are multiple equilibria with prices along the boundary between the low-price and the medium-price regions.

In the interior of Region 1, the first-order conditions of the medium-price range suggest a pair of prices strictly within the medium price range, and therefore the equilibrium again is again unique. However, to make sure the first-order conditions of the medium-price region define the global equilibrium, we also need to compare the profits given these prices to the profits from the optimal deviation to the high-price region, which is now not empty. That optimal deviation results in lower profits when \( V > 1 \) and higher profits when \( V < 1 \).

As stated above, when \( V < 1 \), responding to the solution of the first-order conditions of the medium-price region by a price in the high-price region is always better than responding by the optimal price in the medium price region. As we already stated before, a pure-strategy equilibrium does can not exist in the high-price region, and thus for \( V < 1 \), only a mixed strategy equilibrium in prices exists.

To understand the nature of this mixed strategy equilibrium in the \( V < 1 \) region, consider the following process of price adjustment starting from the solution of the first-order equations
in the medium-price region. First, the optimal response to such price is a higher price that put the two prices in the high-price region. The optimal response to that high price is to slightly undercut it. The optimal response to the new price is to again slightly undercut it (still resulting in a pair of prices within the high-price region). The undercutting argument continues until one firm finds it better to charge a much higher price than to undercut (this condition is reached before the prices move to the medium price region). But then the other firm will undercut again and the foregoing analysis repeats. This suggests that for $V < 1$, the equilibrium price is a mixed strategy within high price region. The explicit solution for the mixed strategy equilibrium is not analytically tractable.

We now turn to the implications of Proposition 2. First note that it implies that there is a market for fashion for any $V$ including $V > 2$. When $V > 2$, one could speculate that the low type consumers could just buy both products while the high-type consumers buy one of them even if the editor fully randomize between the two products. This would imply that the high-type consumers are better off not buying anything and the editor would always select neither. However, for a high $V$, the editor is still able to make the products worthwhile to the high-type consumers by choosing them with sufficiently low probability. In equilibrium, although the probability of the editor selecting one of the products tends to zero as $V$ tends to infinity, the market (total demand) for fashion products actually expands, as the increased demand from high-type consumers is more than compensated by the increased demand from the low-type consumers.

Proposition 2 also implies that even without the restriction of symmetric equilibrium, the equilibrium price is non-monotone in $V$ since the unique equilibrium price is decreasing in $V$ in Regions 1 and 4 and is higher at the upper end of Region 1 than at the lower end of Region 4. Since prices are equal to zero when $V = 0$, the equilibrium price first increases, then decreases, then increases again, and finally decreases asymptotically approaching $\sqrt{\alpha} - \alpha$ as $V \to \infty$ (see Figure 2 for an illustration for $\alpha = 1/4$). It is also interesting to observe that among all possible equilibria in Region 2, the symmetric one given in Proposition 2 is the optimal one for the industry in the sense that total industry profit is the highest among all potential equilibria.
However, the symmetric equilibrium is not Pareto optimal. In fact, while the lower-price firm is worse off in any asymmetric equilibrium than in the symmetric equilibrium, the high-price firm is not only better off than the lower-price firm but also better off than it would be in the symmetric case.

Comparing the equilibrium prices to the preferred-to-the-editor outcome, we find that equilibrium prices are lower and more low type consumers buy than what the editor would prefer. This reflects the tension between the editor and the firms’ interests. The editor wants to exclude the low type consumers from matching while firms want to go attract the low-type consumers to increase profit. The equilibrium prices are, however, higher than the socially optimal level. When $V > 1$, the socially optimal outcome requires that the low type consumers be involved in pairing up with high-type consumers to the maximum extend. Yet in equilibrium, high-type consumers have a higher chance of pairing with high-type consumers than the low-type consumers have.

Although the preference structure specified in this model is different in nature from the one used in the classic pricing games, an upward shift in $V$ could still be interpreted as an overall increase in consumer valuation. This increase, however, doesn’t lead to a higher equilibrium price. This is because when the low type consumers’ valuation for signaling ($V$) increases, it becomes more attractive for a firm to attract the low type consumers, and the consumer value of the product declines. In fact, Proposition 2 implies that when $V$ is sufficiently high, the equilibrium demand from the high type consumers decreases but that from the low-type consumers increases.

As Proposition 2 indicates, the profit implications of an increase in low-type consumer valuation $V$ of pairing with high-type consumers are also not straightforward. Equilibrium profit of each of the two firms, increases in $V$ when $1 \leq V \leq V_1$. This is because in that area, although higher $V$ leads to lower price, the larger demand resulting from more low-type consumers buying more than offsets the profit effect of the lower price and overall profit increases. On the other hand, when $V > V_2$, an increase in low type’s valuation for signaling is detrimental to the firms. As low-type consumer valuation for matching with the high-type consumers increases,
it becomes more and more difficult for the editor to keep low-type consumers from buying and as a result, low type demand for each product goes up with $V$. This decreases the potential value of the product to either consumer type given any prices, and results in the optimal price declining so that the price decline effect dominates the market expansion effect and thus the profit of each firm decreases. In Region 3, the equilibrium price increases in $V$ but total demand decreases in $V$. These two opposing forces first drive profits up and then down.

5.2 Role of the Editor

To understand the role played by the editor in the fashion industry, let us compare the above results to the results we would obtain if the editor would not exist in the model. If the editor does not exist, the model is than the same as the main model except that Stage 2 does not exist. The timeline of the game then becomes the following:

1. Firms 1 and 2 simultaneously set prices.
2. All consumers make purchase decisions.
3. The “matching game”, in which consumers with the same product are randomly matched into pairs and utility of matching is realized.

If we keep the assumption that using a product not used by anybody else results in zero payoff, then nobody buying any product is always an equilibrium for any $V$ and any prices. When $V < 1$, all high type consumers buying either product is an equilibrium of the consumer choice subgame. Such a game has numerous equilibria and the market outcome is unpredictable.

In light of this issue, we consider a slightly modified assumption that there is always a small fraction (zero mass) of high type consumers who will use Product $k$, $k = 1, 2$, in the matching game.\textsuperscript{14} This assumption ensures that a consumers will always have a match regardless of the product he/she uses in the matching game. With this assumption, we have the following result

Lemma 1. In the model with no editor,

1. When $V > 1$, in equilibrium, demand for either product is zero.
2. When $V < 1 - \alpha$, the unique equilibrium prices are $p_1 = p_2 = V$. In equilibrium, all high-type consumers and only they buy one of the products. Any split of market share between the two products is an equilibrium outcome.

\textsuperscript{14}One possible explanation is that these consumers are not strategic. Another explanation is that these consumers are paid by the firm to wear the product.
Proof. See Appendix.

In other words, in the absence of the editor, the market either does not exist, or the market shares and profits are not predicted by the equilibrium. Therefore, we have the following result as a corollary to the above lemma and Proposition 2.

**Proposition 3.** While in the absence of editor the market for fashion may only exist when \(0 < V < 1\), with editor, the market for fashion will exist for any \(V > 0\).

*Proof.* See Appendix.

In the no-editor model, market doesn’t exist when the low type’s valuation for signaling is high due to the difficulty of keeping the low type consumers from buying the same product as the high-type consumers. The existence of an editor solves this problem. Thus, for \(V > 1\), the editor creates the market for fashion given the assumption that the high-type consumers have a superior to the low-type consumers’ connection with the editor. To take advantage of this difference, the editor must not choose her recommendations deterministically. Indeed, while too wide difference in prices could technically counter-balance the above incentive to randomize, it turns out that in equilibrium, the prices are neither high enough nor sufficiently different from each other.

**Proposition 4.** In a pure-strategy equilibrium, there will always be randomness in the selection of fashion hits.

*Proof.* Directly follows from Proposition 2.

Since the high type consumers follow the editor’s choice, the consequent fashion hit will also appear random. This is consistent with the popular observation that fashion hits materialize randomly. Our model suggest however, that the hits are not arbitrary in the sense that they are random because the editor intentionally randomizes the coordinated choice of the high-type consumers according to a uniquely-defined in the equilibrium rule. If the editor decision were deterministic, the low type consumers would be able to expect the outcome and purchase the
product that is slated to become the “it”. The outcome would be the same as in the case without
the editor and would not be optimal for the high-type consumers. Thus, the editor does not
follow a deterministic strategy. Since the utility of the high types is directly affected by the
number of low types who use the same product, she attempts to reduce the low-type consumer
purchases of the hit product through randomizing. This exact formula for this randomization
is unique and depends on the product prices. In other words, there is a method behind the
seeming madness of fashion whimsy. The randomness of fashion hits is a result of consumers’
need for signaling and editor’s picks on behalf of the high-type consumers.

5.3 Effect of Competition

Let us now examine the effect of competition on the fashion industry. To do this, we consider a
monopoly firm offering two products in the same market and compare the results to the duopoly
market above.

Since this game following the price-setting stage is the same as one in the main model,
the analysis of the consumers’ purchase decision and editor’s recommendation strategy applies
in this game. The only difference is that when solving for the optimal prices in Stage 1, the
objective function for the firm is the sum of the two products’ profits rather than the profit
from just one product. This leads to the following results.

Proposition 5. If both products are managed by a monopolist, when $1 \leq V < 2 + \frac{\alpha}{2} - \frac{\sqrt{\alpha^2 + 8\alpha}}{2}$,
the monopolist sets prices $p_1 = p_2 = \frac{\alpha V}{2-V}$, which are lower than in the competitive market, and
the editor randomizes between the two products with equal probability. The profit of the monopoly
firm is $\frac{\alpha V (1-\alpha-V)}{2-V}$. When $V \geq 2 + \frac{\alpha}{2} - \frac{\sqrt{\alpha^2 + 8\alpha}}{2}$, the optimal monopoly price is the same as in the
competitive case.

Note that in this model, the monopoly charges such prices that the demand from low type
consumers for both products is positive. We know from Proposition 1 that an increase in the
own price of a product will lead it to be chosen with higher probability. Since the editor has to
tradeoff between the two products, this will inevitably lead to the other product being with lower
probability and earning lower profit. This will hurt the total profit of the monopoly firm and
the monopoly finds it optimal to keep prices at a lower level. This is the intuition for the result that when \( V \) is small and therefore prices are in Region 1, competition drives up the equilibrium prices. One may find this result surprising since normally one would expect competing firms’ incentive to increase own market share to lead to a lower price. In this model, on the contrary, it is in the best interest of the industry to keep prices lower than the competitive incentives would suggest, since an increase of one product’s price hurts the profitability of the other product. Note that compared with the competitive case, the monopoly case is farther away from what is desired by the editor \( (p_1 = p_2 = V/2) \), but if \( V > 1 \), it is closer to the socially optimal outcome \( (p_1 = p_2 = V/2) \).

Another somewhat surprising observation from the comparison of the results in the monopoly and competitive cases, is that they lead to exactly the same outcomes in Regions 2, 3 and 4. In other words, when \( V \) is relatively large, firms essentially do not compete in prices. The intuition for this is that when \( V \) is large, the only relevant case is the low-price region. In this case, the revenue from a product does not depend on the price of the other product. Therefore, firms are effectively competing with the “neither” option instead of competing with each other, and therefore the outcomes of the competitive and industry-maximizing strategies are the same.

6 Discussion and Conclusion

It is a common adage that the fashion industry is fast-changing and highly unpredictable. While in many markets the consumer value of a product or service is slowly changing through time, the value of a potentially fashionable item may drastically change from month to month. Understanding this industry is further complicated by the significant influence of fashion editors and other opinion leaders. In this paper, we presented an equilibrium analysis of competitive and intrinsically random fashion market explicitly modeling the role of fashion editor. As a result, we are able to understand the role and regularities of the random fashion changes and suggest how firms’ strategy in a fashion market with the editor or another coordinating force is different from more steady status good markets.

The first consequence of the editor’s role is that in equilibrium, fashion selection is always
probabilistic as opposed to deterministic. This is because when deterministic strategy could work, one firm undercutting the other’s price by a small amount would lead it to gain market share significantly larger than a half. This would not happen without the editor: as Pesendorfer (1995) and Bagwell and Bernheim (1996) show, in the absence of the editor, there is a point reducing price below which leads to loss of all sales since the product can no longer signal status. However, when the editor is present and the price is reduced below that point, the status value of the product is preserved by the editor placing the probability on its selection below one.

Another but related implication of the role of the fashion editor is that competition leads to prices that result in a strictly positive expected demand from the low-type segment. At first glance, this may seem counter-intuitive, since given the assumed two-segment structure of the market, the preferred solution for the high-type consumers is the cheapest status good that would fully separate them from the low-type consumers (Pesendorfer 1995, Bagwell and Bernheim 1996). Within our model, this outcome is achievable when $V < 2 \left(\frac{V}{2}\right)$. Since one could expect firms to try to raise prices, one can understand how prices could become higher. Why would the presence of the editor who acts on behalf of high-type consumers lead instead to lower prices then the best for high-type consumers? The intuition for our result is that if one of the two prices is reduced, the editor can not commit to choose the higher priced product with higher probability. If low-type consumers believed that the editor would do so and abstained from buying, the editor would then choose the cheaper product. To prevent the editor from such opportunistic behavior, the equilibrium low-consumer demand for the cheaper product must be positive, and the result is that both products receive positive expected demand.

We also find that with the editor, the fashion market always exists. This is in contrast to the result that market may not exist in the no-editor model when low type’s valuation for signaling is too large. The idea behind this result is again that by not choosing either product most of the time, the editor allows the products to still play a role of status symbols in the remaining instances.

While supporting the notion that fashion hits are random, our model implies that fashion hits are not random in nature, but according a pattern determined by the editor and affected
by firms’ pricing strategies, both of which are uniquely defined in equilibrium. In particular, we show that whenever the demand from low type consumers for a product is positive, an increase in the price of that product always leads to higher probability of being chosen by the editor and lower demand from the low type consumers. This suggests that for the editor’s decision of which product to pick, reducing demand of the low type consumers is a more important consideration than the price of the product.

Combining the result about the uniqueness of the editor’s equilibrium strategy with the existence of the market, one could say that while appearing arbitrary, the editor in fact plays a stabilizing role in the fashion industry. Understanding the process of the fashion hit selection allows us to further consider and predict firms’ profits and their profit-maximizing strategies. For example, we observe that in some range of the low-type consumers’ valuation for signaling, competition between firms drives up market price. Furthermore, the equilibrium price zigzags in the low-type consumers’ valuation for signaling when the low-type consumer valuation for signaling increases from zero to infinity, as firms trade-off between attracting the increased demand of the low-type consumers and increasing the expected status of their products. In particular, the equilibrium price is also non-monotone in the fraction $\alpha$ of the high-type consumers. The equilibrium profit, first increases and then decreases in $V$, implying that there is an optimal-for-the-industry value of low-type consumer valuation of signaling. When $V$ is large enough, the monopoly case and competitive case lead to the same equilibrium outcomes, since firms are competing with the outside option rather than with each other. As $V$ tends to infinity, the equilibrium price asymptotes to $\sqrt{\alpha} - \alpha$ and profits asymptote to $(\sqrt{\alpha} - \alpha)^2$, so that for large $V$, the optimal-for-the-industry amount of high-type consumers in the market approaches $1/\sqrt{2}$. 
Notes about Figure 1:

1) The line ACB will always be below ADB as long as the medium-price and high-price area exist.

2) When prices are symmetric, $P_1 = P_2 = \alpha V/(2-V)$ marks the boundary between the low-price and medium-price area, and $P_1 = P_2 = V/2$ separates the medium- and high-price area.

3) As $V$ increases, line ACB and ADB will shift upward and rightward, but will always be symmetric around the 45 degree line. Also, the low-, medium- and high-price area will all exist as long as $V<2-2\alpha$. When $V>2-2\alpha$, both the medium- and high-price area disappear. Also, for $V<1-\alpha$, there is an additional “very high prices” area above the high-price area.
Figure 2: change of equilibrium price with $V$ when $\alpha=1/4$

Figure 2: change of equilibrium profit with $V$ when $\alpha=1/4$
Appendix

Proof of Proposition 1

We’ll first show that when \( p_1 < 1 - p_1/V + p_1 \) and Equation (2) in the main text is satisfied, the editor must choose both products with positive probability.

When \( p_k < 1 - \alpha \), the editor must choose at least one product with positive probability. Suppose the editor chooses neither with probability 1, then her is equal to \( \alpha \) and the low type will buy neither product. Her payoff of picking Product 1 now is \( 1 - p_k \). Since \( p_k < 1 - \alpha \), the editor will deviates to picking Product \( k \).

If in the equilibrium, the editor picks a product with probability 1, then it must be Product 2, the cheaper product. It has been shown in the main text that in the equilibrium, it’ll never be the case that all low types buy a product. We’ll now show that it cannot be either that a fraction of low types buy that product, provided \( p_1 < 1 - p_1/V + p_1 \) holds. Suppose the contrary is true, then the low types must be indifferent between buying and not buying, which implies \( V\alpha/(\alpha + x_2) - p_2 = 0 \). The editor’s payoff then is \( \alpha/(\alpha + x_2) - p_2 = \frac{(1-V)p_2}{V} \). If she deviates to Product 1, her payoff would be \( 1 - p_1 \). When \( p_1 < 1 - p_1/V + p_1 \) holds, \( \frac{(1-V)p_2}{V} < 1 - p_1 \) and the editor will deviate. So this case won’t happen in the equilibrium.

Now suppose the editor picks Product 2 with probability 1 and no low types buy that product. Since the low types don’t buy, it implies that \( p_1 \geq p_2 \geq V \). However, this contradicts the condition \( p_1 < \frac{V - (1-V)p_1}{V} \). So it won’t happen in the equilibrium, either. Hence in the equilibrium, the editor must choose both products with positive probability.

Next we’ll show when the editor chooses both products with positive probability, only the three cases listed in Proposition 1 (Item 1-3) are possible.

When the editor chooses all three options with positive probability, only one case is possible: a fraction of low types buy each product. If all low types buy a product, high type’s payoff must be smaller than \( \alpha \) and the editor will deviate to neither. If no low type buys a product, then high type’s payoff must be greater than \( \alpha \) and the editor will not choose neither. So low type’s strategy on both products must be mixed.
When the editor randomizes between the two products only, all low types cannot buy a product, or the editor will deviate to neither. Also, demand from the low type for both products cannot be 0 at the same time unless prices are equal, or the editor will deviate to the cheaper product. When the low type only buys one product, then it must be the cheaper product. These restrictions imply that when the editor randomizes between product 1 and 2, only two possible cases are possible: a fraction of low types buy each product, or a fraction of low types buy Product 2 only.

Next we’ll prove the four items in the proposition

**Proof of item 1**

The editor must be indifferent between picking Product 1, Product 2 and neither. Her payoff is \( \frac{\alpha}{(\alpha + x_k)} - p_k \) when picking Product k, \( k=1,2 \) and \( \alpha \) when picking neither. This implies \( \frac{\alpha}{(\alpha + x_k)} - p_k = \alpha \).

The indifference conditions of the low types lead to the following equations:

\[
V\delta_k \frac{\alpha}{\alpha + x_k} - p_k + (1 - \delta_1 - \delta_2)\alpha = (1 - \delta_1 - \delta_2)\alpha
\]

Solving these equations leads to the results given in Table 1.

The solution for \( x_1, x_2, \delta_1 \) and \( \delta_2 \) need to satisfy the boundary conditions \( 0 \leq x_k \leq 1 - \alpha, \delta_k > 0 \) and \( 1 - \delta_1 - \delta_2 > 0 \). When condition (2) in the main text is satisfied, \( 0 < x_k < 1 - \alpha \). Since \( p_k > 0, \delta_k > 0 \). In addition, when the condition specified in Item 1 holds, \( 1 - \delta_1 - \delta_2 > 0 \) is satisfied.

Next we’ll check for deviations. The editor and the low types won’t deviate by assumption since they are indifferent between all the options. The high types won’t deviate either since the editor is maximizing the payoff of the high type consumers. This concludes the proof.

**Proof of item 2**

We have shown that when \( p_1 < 1 - p_1/V + p_1 \) is satisfied, only the three cases listed in the proposition are possible. So when the conditions for the other two cases are not satisfied, we’ll
end up in this case. The editor must be indifferent between picking Product 1 and Product 2, which leads to the condition

$$\frac{\alpha}{\alpha + x_1} - p_1 = \frac{\alpha}{\alpha + x_2} - p_2$$

The indifference condition of the low types implies

$$V\delta_k \frac{\alpha}{\alpha + x_k} - p_k = 0$$

Finally, the editor will only pick the two products with positive probabilities: $\delta_1 + \delta_2 = 1$. Solving these 4 equations yields the results given in Table 1.

**Proof of item 3**

The indifference condition of the editor suggests

$$\frac{\alpha}{\alpha + x_1} - p_1 = \frac{\alpha}{\alpha + x_2} - p_2$$

The indifference condition of the low types implies.

$$V\delta_2 \frac{\alpha}{\alpha + x_2} - p_2 = 0$$

Low type demand for Product 1 is 0: $x_1 = 0$. The editor never choose neither with positive probability: $\delta_1 + \delta_2 = 1$. Solving these 4 equations gives rise to the results in Table 1.

We’ll now check the boundary conditions for $x_2$, $\delta_1$ and $\delta_2$. As long as $p_1 > p_2$, $0 < x_2 < 1 - \alpha$. $\delta_2 > 0$ is always satisfied. When the condition specified in Item 3 is satisfied, $\delta_1 > 0$.

Next we’ll check for deviations. The payoff to the editor is $1 - p_1$ in the equilibrium versus $\alpha$ when she picks neither. When Equation (2) holds, the editor will not deviate. For the low types, the payoff of buying Product 1 is $V\delta_1 - p_1$ and the payoff of buying neither is 0. When the condition specified in Item 3 is satisfied, $V\delta_1 - p_1 < 0$ so the low types won’t deviate.

**Proof of item 4**

When $p_2 \geq V$, the condition specified in the proposition is automatically satisfied. Equation (2) implies $p_1 \geq p_2 \geq V$. The low types will not buy any product regardless of the editor’s choice. So the editor will pick Product 2 with probability 1 and $x_2 = 0$. 

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When $p_2 < V$ and $\delta_2 = 1$, a positive fraction of low types will buy Product 2. The indifference condition of the low types implies:

$$V * \frac{\alpha}{\alpha + x_2} - p_2 = 0$$

which yields $x_2 = \frac{\alpha(V-p_2)}{p_2}$. It remains to check for deviations. High type (and the editor)'s payoff is $\frac{(1-V)p_2}{V}$. The payoff to the editor by deviating to Product 1 is $1 - p_1$. When the condition specified in Item 1 is satisfied, $\frac{(1-V)p_2}{V} > 1 - p_1$, so the editor won’t deviate. This leads to the summary of results in Table 1.

**Proof of Proposition 2**

The full equilibrium of the game is in Area 1, 2 or 3 (corresponding to item 1, 2 and 3 in Proposition 1), in which the editor chooses both products with positive probability.

**Low price case** With the profit function of the two firms given in Table 2, we have

$$\frac{\partial \pi_i}{\partial p_k} = p_k^2 + 2\alpha p_k + \alpha V - V p_k^2 - 2\alpha V p_k - \alpha^2 V$$

When $V \leq 1$, $\frac{\partial \pi_i}{\partial p_k}$ is always positive and firms always benefit from increasing price. So an equilibrium couldn’t exist in this case. When $V > 1$, $\frac{\partial \pi_i}{\partial p_k}$ is positive for small $p_k$ but negative for large $p_k$. The optimal price, denoted by $p_{\text{low}}$, is given by

$$p_{\text{low}} = \frac{\alpha - \alpha V + \sqrt{\alpha^2 + \alpha V^2} - \alpha V - \alpha^2 V}{V - 1}$$

So $p_1 = p_2 = p_{\text{low}}$ is an candidate for the equilibrium.

**Medium price case** The profit function of the two firms is given in Table 2. An equilibrium could be found by solving the first-order conditions of the two firms simultaneously, which gives

$$p_1 = p_2 = p_{\text{med}} = \frac{V(V-2))}{V+2}$$

This is also a candidate for the equilibrium of the game.

**High price case** The profit functions are given in Table 2. Since the profit functions are not symmetric, we need to examine them separately. For product 2, $\frac{\partial \pi_2}{\partial p_2}$ is always positive, so it’s
always beneficial to increase price. For product 1, $\frac{\partial \pi_1}{\partial p_1}$ is positive for small $p_1$ and negative for large $p_1$. the optimal price is found by solving for the first-order condition $\frac{\partial \pi_1}{\partial p_1} = 0$. Denote this optimal price by $p_{high}$.

$$p_{high} = \frac{V + Vp_2 - \sqrt{Vp_2 + Vp_2^2}}{V}$$

The equilibrium cannot exist in the high price case. However, $p_{high}$ is useful when checking for deviations.

In summary, $p_1 = p_2 = p_{low}$ and $p_1 = p_2 = p_{med}$ could be the equilibrium if they satisfy the boundary conditions. If the boundary conditions are violated, then the prices at the boundary could be the equilibria of the game.

When $p_1 = p_2 = p_{med}$, the profit function for each firm is

$$\pi_1 = \pi_2 = \pi_{med} = \frac{\alpha V^2}{V + 2}$$

When one firm charges $p_{med}$ and the other firm responds by the “high-optimal price”, then the deviating firm will get

$$\pi_{meddev} = \frac{\alpha(2 - V - \sqrt{2 - V \sqrt{2 - V^2 + 3V}})(V^2 - 3V - 2 + \sqrt{2 - V \sqrt{2 - V^2 + 3V}})}{\sqrt{2 - V \sqrt{2 - V^2 + 3V}}(2 + V)}$$

$\pi_{med} < \pi_{meddev}$ when $V < 1$ and $\pi_{med} > \pi_{meddev}$ when $V > 1$. $\pi_{med} = \pi_{meddev}$ when $V = 1$. So $p_1 = p_2 = p_{med}$ couldn’t be the equilibrium when $V < 1$.

When $p_1 = p_2 = p_{med}$, low type consumer’s demand for each product is $x_1 = x_2 = \frac{\alpha(3V-2)}{2(2-V)}$ and high type consumers’ equilibrium payoff is $\frac{(V-2)^2}{V+2}$. Three boundary conditions need to be satisfied.

1. $x_k > 0$. This is satisfied when $V \geq 1$.
2. $x_k \leq 1 - \alpha$.
   This is satisfied for $V < \frac{2(2-\alpha)}{2+\alpha}$.
3. Incentive constraint of the editor. Need to check that the editor’s equilibrium payoff is greater than $\alpha$, which requires

$$V < V_1$$

where $V_1$ is defined in the Proposition 2. This is a stronger condition than the previous one.
This concludes that when \(1 \leq V \leq V_1\), \(p_1 = p_2 = p_{med}\) is the equilibrium of the game, which proves Item 1 in the proposition.

\(p_1 = p_2 = p_{low}\) could also be an equilibrium of the game. In order for this equilibrium to hold, the boundary condition in Item 1, Proposition 1 must be satisfied. We reprint it here for convenience.

\[
\alpha(1-V)(p_1 + p_2) + (2-V)p_1p_2 < \alpha^2V
\]

When \(V > 2 - 2\alpha\), this condition is satisfied for any \(p_1, p_2 \leq 1 - \alpha\). When \(V < V_2\), \(p_{low} < 1 - \alpha\). So naturally when \(V > \max\{2 - 2\alpha, V_2\}\), \(p_1 = p_2 = p_{low}\) is the equilibrium of the game. When \(2 - 2\alpha < V < V_2\), \(p_1 = p_2 = 1 - \alpha\) is the equilibrium instead of \(p_1 = p_2 = p_{low}\). This proves Item 3 in the proposition.

When \(V_2 \leq V < 2 - 2\alpha\), \(p_1 = p_2 = p_{low}\) satisfies the boundary condition, but we still need to check for deviations. Suppose one firm (say firm 1) charges \(p_{low}\) and the other (firm 2) decides whether to deviate. We could be in the low, medium or high price case depending on the price of firm 2. By definition of \(p_{low}\), deviation within the low price case is never profitable. The cutoff price between the low price case and medium price case is given below

\[
c_1 = \frac{\alpha(\alpha - \alpha V + (1-V)\sqrt{\alpha(\alpha + V^2 - V - \alpha V)})}{\alpha V - \alpha - (2-V)\sqrt{\alpha(\alpha + V^2 - V - \alpha V)}}
\]

If \(p_2 \leq c_1\), we are in the low price case. If \(p_2 > c_1\), we are in the medium price case. \(\frac{\partial \pi_2}{\partial p_2}\) when \(p_2 \geq c_1\), so profit decreases in the medium price case and deviation within this area cannot be profitable. In the high price area, the most profitable deviation is to the “high optimal” price. However, the deviation profit is always smaller than the equilibrium profit, so the firm won’t deviate. This suggests whenever \(V \geq V_2\), \(p_1 = p_2 = p_{low}\) is the global equilibrium of the game. This also proves Item 4 in the proposition.

Since \(V_1 < V_2\) always holds, \([1, V_1]\) and \([V_2, \infty)\) never intersects each other, so \(p_1 = p_2 = p_{low}\) and \(p_1 = p_2 = p_{med}\) cannot be the equilibrium at the same time. This suggests they are the unique equilibrium of the game in their respective region.

When \(V_1 < V < \min\{2 - 2\alpha, V_2\}\), neither \(p_1 = p_2 = p_{med}\) nor \(p_1 = p_2 = p_{low}\) is the equilibrium since the boundary conditions are violated. Under this condition, the prices at the
boundary between the low prices and medium price case are the equilibria of the game. In particular, there is a symmetric pure-strategy equilibrium in which

\[ p_1 = p_2 = p_{\text{sym}} = \frac{\alpha V}{2 - V}, \quad \pi_1 = \pi_2 = \pi_{\text{sym}} = \frac{\alpha V (\alpha + V - 2)}{2(\alpha + V - 2)} \]

Without loss of generality, assume that firm 1 charges \( p_{\text{sym}} \) and firm 2 decides whether to deviate. For firm 2, \( \frac{\partial \pi_2}{\partial p_2} > 0 \) when \( p_2 < p_{\text{sym}} \) (low price case) and \( \frac{\partial \pi_2}{\partial p_2} < 0 \) when \( p_2 > p_{\text{sym}} \) (medium price case). So firm 2 won’t deviate to these two areas. It remains to check for deviations to the high price case. The high optimal price for firm 2 is given by

\[ p_{\text{symdev}} = \frac{2 + \alpha V - V - \sqrt{\alpha(2 + \alpha V - V)}}{2 - V} \]

And the deviation profit for firm 2 is

\[ \pi_{\text{symdev}} = \frac{(\alpha - \sqrt{\alpha(2 + \alpha V - V)})(V - \alpha V - 2 + \sqrt{\alpha(2 + \alpha V - V)})}{(2 - V)\sqrt{\alpha(2 + \alpha V - V)}} \]

However, the high optimal price may not be feasible. If \( p_{\text{symdev}} > 1 - \alpha \), the deviation price becomes \( 1 - \alpha \). Let \( c_2 = \frac{2(1 - 2\alpha)}{1 - \alpha^2} \). Then if \( V < c_2 \), \( p_{\text{symdev}} < 1 - \alpha \) and vice versa.

Denote the profit of firm 2 if it deviates to \( 1 - \alpha \) by \( \pi_{\text{symdev2}} \). Then \( \pi_{\text{symdev2}} = \frac{\alpha(1 - \alpha)}{2} \). \( \pi_{\text{sym}} - \pi_{\text{symdev2}} \) is concave in \( V \) and \( \pi_{\text{sym}} - \pi_{\text{symdev2}} > 0 \) at both \( V = c_2 \) and \( V = V_2 \). This implies deviation to \( 1 - \alpha \) is never profitable. For the same reason, deviation to the high optimal price is not profitable either whenever it’s feasible. This proves \( p_1 = p_2 = p_{\text{sym}} \) is the equilibrium.

Consider the case where \( p_1 \) and \( p_2 \) change by a small amount from \( p_{\text{sym}} \), but still at the boundary between the low price and medium price case. Since the profit functions and their derivatives are all continuous in price, when the changes are small enough, the equilibrium profit is still larger than the deviation profits. This suggests the game has multiple equilibria in this region, which proves Item 2 in the proposition.

**Proof of Lemma 1**

**Proof of item 1 (the case of \( V > 1 \))**

When \( V > 1 \), the low types will buy whenever the high types buy. Need to show that nobody buying any product (except for possibly a zero mass of consumers) is always the equilibrium
of the subgame. Consider the following situation: a zero mass of both high type and low type consumers buy Product \( k \), \( k = 1, 2 \). Among them, the proportion of high type consumers is \( \alpha + p_k/V \). Other consumers don’t buy any product. In this case, the equilibrium payoff is \( \alpha \) to the high type consumers and \( \alpha V \) to the low types. Deviation payoff of buying Product \( k \) is \( \alpha + p_k/V - p_k \) for the high types and \( \alpha V \) for the low types. Since \( V > 1 \), \( \alpha + p_k/V - p_k < \alpha \) and so the high types won’t deviate. The low types won’t deviate either. This concludes that nobody buying any product is indeed the equilibrium of the game.

**Proof of item 2 (the case of \( V < 1 - \alpha \))**

When analyzing the consumer choice subgame, we still make the assumption that \( p_1 \geq p_2 \).

When \( p_1 > p_2 \geq V \), all but a zero mass of high type consumer will buy Product 2. The low type consumers will buy neither product, since the prices exceed their valuation for signalling.

When \( p_1 = p_2 \geq V \), the low types still won’t buy any product. All high type consumers will buy one of the products, but any split of market share between the two products could be an equilibrium.

When \( V \geq p_1 > p_2 \), consider the following strategy: all but a zero mass of high type consumers buy Product 1. A zero mass of non-strategic high type consumers buy Product 2. Low type’s demand for Product 1, \( x_1 \), is equal to \( \alpha V/p_1 - \alpha \) if \( p_1 \geq \alpha V \) or \( 1 - \alpha \) if \( p_1 \leq \alpha V \). \( x_2 = 0 \), but there is a zero mass of low type consumers buying Product 2 such that the proportion of high type consumers among all Product 2 users is equal to \( \alpha V/p_1 - \alpha \). Under this strategy, \( U_l(1) = U_l(2) = \frac{\alpha}{\alpha + x_1} - p_1 \) so they won’t deviate. For the high type consumers, \( U_h(1) - U_h(2) = (p_1 - p_2)(1/V - 1) > 0 \), so they won’t deviate either. Therefore the foregoing strategy is the equilibrium of the consumer choice subgame.

When \( p_1 \geq V > p_2 \) and \( p_1 < 1 + (1 - 1/V)p_2 \), consider the following strategy: all but a zero mass of high type consumers buy Product 1. A zero mass of them buy Product 2. \( x_1 = x_2 = 0 \), but there is a zero mass of low type consumers buying Product 2 such that the proportion of high types consumers is equal to \( p_2/V \). Under this strategy, \( U_l(2) = U_l(0) = 0 > U_l(1) \), so the low types won’t deviate. For the high types, \( U_h(1) - U_h(2) = 1 + (1 - 1/V)p_2 - p_1 > 0 \), so they
won’t deviate either. This means this is the equilibrium of the subgame.

When \( p_1 \geq V > p_2 \) and \( p_1 > 1 + (1 - 1/V)p_2 \), following similar argument, it could be shown that the following strategy constitutes an equilibrium: all but a zero mass of high type consumers buy Product 2. A zero mass of them buy Product 1. Low type’s demand for Product 1 is zero and that for Product 2 is equal to \( \alpha V/p_2 - \alpha \) if \( p_2 > \alpha V \) and \( 1 - \alpha \) otherwise.

Next we’ll show that the unique equilibrium of this game is \( p_1 = p_2 = V \). Conjecture an equilibrium of the game \( (F_1(p_1), F_2(p_2)) \), where \( F_k(p_k) \) denotes the distribution of prices that firm \( k \) adopts in the equilibrium. Let \( \overline{p}_k \) and \( \underline{p}_k \) denote the upper and lower bound of the support of \( F_k(p_k) \).

Note that any price \( p_k < V \) is dominated by \( p_k = V \). This is because for \( p_j \neq V, j = 3 - k \), \( p_k = V \) guarantees getting the whole market, while for \( p_j = V \), any price \( p_k \neq V \) gives zero market share and profit. This leads to the result that \( p_k \geq V \).

Next, suppose \( p_k > V \). Further, suppose there is a mass point at \( p_k = \overline{p}_k \). Then the best response of firm \( j \) requires that \( \overline{p}_j \leq p_k \) and no mass point at \( p_j = \overline{p}_j \), since any price \( p_j > \overline{p}_k \) guarantees zero profit. Then the optimal strategy of firm \( k \) is to remove the mass point at \( p_k = \overline{p}_k \) and just undercut firm \( j \). This contradiction means there is no mass point at \( \overline{p}_k \), \( k = 1, 2 \). Also, \( 1 - F_k(\overline{p}_j) = 0 \) must be satisfied, since any price higher than \( \overline{p}_j \) gives zero profit. Without loss of generality, we assume that \( \overline{p}_1 = \overline{p}_2 \). Given this, \( p_k = \overline{p}_k \) is dominated by \( p_k = V \), since the former gives zero profit while the latter guarantees positive profit. This means \( \overline{p}_k > V \) couldn’t hold in the equilibrium and we must have \( \overline{p}_k \leq V \). Combined with the result that \( p_k \geq V \), we have \( \overline{p}_k = \overline{p}_k = V \) for \( k = 1, 2 \). So \( p_1 = p_2 = V \) is the unique equilibrium of the game.

**Proof of Proposition 3**

From Proposition 1, case (4) couldn’t happen with positive probability in the equilibrium since at least one of the firms is getting zero sales and profit. For all the other cases, the sales and profit of both firms is positive. So in the equilibrium, both firms must enjoy positive sales and profit.
Proof of Proposition 5

We’ll first explore the change of total monopoly profit with respect to prices.

Low price case In the low price case, the profit of the individual product doesn’t depend on the price of the other product. So maximizing total profit is the same as maximizing the profit for individual products. From the proof of Proposition 1, \( p_1 = p_2 = p_{\text{low}} \) is the optimal prices provided they satisfy the boundary condition. If it doesn’t satisfy the boundary condition, the optimal prices will be on the boundary. Let

\[
p_{\text{cut}}(p_2) = \frac{\alpha(V\alpha + p_2V - p_2)}{2p_2 + \alpha - p_2V - V\alpha}
\]

Then any pair of prices \((p_{\text{cut}}(p_2), p_2)\) in on the boundary of the low-price case and the medium-price case. The total profit for charging \((p_{\text{cut}}(p_2), p_2)\) is

\[
\pi_{\text{Boun}}(p_2) = \frac{\alpha(2V\alpha p_2 - p_2V^2\alpha + \alpha V^3 - V^2\alpha - 2p_2V^2 - p_2^2V^2 + p_2V^3 - 2p_2^2 + 3p_2^2V)}{V(V\alpha + Vp_2 - \alpha - 2p_2)}
\]

with

\[
\frac{\partial \pi_{\text{Boun}}}{\partial (p_2)} = \frac{\alpha(p_2 + \alpha)(V - 1)(2 - V)(p_2V + \alpha V - 2p_2)}{V(V\alpha + p_2V - \alpha - 2p_2)^2}
\]

\(\frac{\partial \pi_{\text{Boun}}}{\partial (p_2)} > 0\) when \(p_2 < \frac{\alpha V}{2 - V}\) and \(\frac{\partial \pi_{\text{Boun}}}{\partial (p_2)} < 0\) otherwise, where \(\frac{\alpha V}{2 - V}\) is the symmetric price on the boundary. This suggests profit of the monopoly firm is maximized when price are symmetric on the boundary, or at

\[
p_1 = p_2 = p_{\text{sym}} = \frac{\alpha V}{2 - V}
\]

It has already been identified in Proposition 1 the condition for \(p_1 = p_2 = p_{\text{low}}\) to hold. Combining all discussions, we can conclude that for the low-price case, profit is maximized at \(p_1 = p_2 = p_{\text{sym}}\) when \(V < v_2\) and \(p_2 = p_2 = p_{\text{low}}\) when \(V \geq v_2\). The profit at \(p_2 = p_2 = p_{\text{sym}}\) is given by

\[
\pi_{\text{optboun}} = \frac{\alpha V(2 - \alpha - V)}{2 - V}
\]

“Medium prices” case The total profit of the monopoly is given by

\[
\pi_{\text{Med}} = \frac{\alpha[(p_1 + p_2)(1 - V) + 2V^2 - \sqrt{(p_1^2 + p_2^2)(V - 1)^2 + 2p_1p_2(1 + 2V - V^2)}]}{2V}
\]

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It could be shown that $\frac{\partial \pi_{\text{med}}}{\partial p_1} < 0$ and $\frac{\partial \pi_{\text{med}}}{\partial p_2} < 0$ for $V < 2$. So the monopoly firm will lose profit by raising any of the prices. Then the optimal prices in the medium-price case is also on the boundary of the low-price and medium-price case. Following the same discussion as in the previous section, the optimal prices will be the same as those in the low-price case.

“High prices” case In the high price case, total profit of the two products is given by

$$
\pi_{\text{High}} = \frac{\alpha(p_1^2V + p_2^2V + p_1p_2 - p_2 - p_1V - 2p_1p_2V)}{V(p_1 - p_2 - 1)}
$$

and

$$
\frac{\partial \pi_{\text{High}}}{\partial p_1} \propto (p_2^2V + 3p_2V - 2p_1p_2V + V - 2p_1V + p_1^2V - p_2)
$$

The right-hand side decreases in $p_1$ and is positive at $p_1 = 1 - \alpha$ when $V > 1$. So raising the price of the more expensive product is always desirable. In order to maximize total profit, the monopoly will set $p_1 = 1 - \alpha$.

$$
\frac{\partial \pi_{\text{High}}}{\partial p_2} \propto (p_1 - 2p_2 + 2p_1p_2 - p_1^2 - p_2)
$$

The right-hand side decreases with $p_2$. So for a given $p_1$, profit first increase and then decreases with $p_2$. The optimal $p_2$ is given by

$$
p_{\text{lowopt}}(p_1) = p_1 - 2 + \sqrt{1 - p_1}
$$

When $p_1 = 1 - \alpha$, $p_{\text{lowopt}}(1 - \alpha) = \sqrt{\alpha} - \alpha$. We need to check whether this price satisfies the boundary conditions for the high-price case.

When $p_1 = 1 - \alpha$, in order for the high-price case to hold, $p_2 > \frac{\alpha(\alpha + V - 1)}{2 - \alpha - V}$ must be satisfied. Comparing these two values, we found that $\sqrt{\alpha} - \alpha$ satisfies the boundary condition when $V < 2 - \alpha - \sqrt{\alpha}$, but is out of bound for $V > 2 - \alpha - \sqrt{\alpha}$.

To conclude the high-price case, when $V < 2 - \alpha - \sqrt{\alpha}$, the optimal prices are $p_1 = 1 - \alpha, p_2 = \sqrt{\alpha} - \alpha$ and the optimal profit is

$$
\pi_{\text{opt high}} = \frac{\alpha(2V + 2\sqrt{\alpha} - 1 - \alpha - 2V\sqrt{\alpha})}{V}
$$
when $V < 2 - \alpha - \sqrt{\alpha}$, the optimal prices are $p_1 = 1 - \alpha, p_2 = \frac{\alpha + V - 1}{2 - \alpha - V}$ and the optimal profit is

$$\pi_{opt\text{high}2} = \frac{\alpha(V^3 + V\alpha^2 + 2V^2\alpha - 2\alpha^2 + 3\alpha - 3V^2 - 4V\alpha + 3V - 2)}{V(\alpha + V - 2)}$$

Direct comparison of $\pi_{opt\text{boun}}$ with $\pi_{opt\text{high}}$ and $\pi_{opt\text{high}2}$ shows that $\pi_{opt\text{boun}} > \pi_{opt\text{high}}$ and $\pi_{opt\text{boun}} > \pi_{opt\text{high}2}$ in their respective region. So the global equilibrium of the monopoly case is $p_1 = p_2 = p_{Sym}$ for $V < v_2$ and $p_1 = p_2 = p_{Low}$ for $V > v_2$. 
References


